Examples of Factoring Quadratic Polynomials with Leading Coefficient Not Equal to One

Written by Victoria Kala
vtkala@math.ucla.edu
Last updated July 20, 2018

Example 1. Factor $6x^2 + 7x - 5$.

Solution. Multiply the leading coefficient and the last term

$$6 \cdot -5 = -30$$

We need to find two numbers that multiply to -30 that add to 7 (the middle term). These numbers are 10 and -3. Replace the middle term with these two numbers:

$$6x^2 + 7x - 5$$

$$6x^2 + (10 - 3)x - 5$$

$$6x^2 + 10x - 3x - 5$$

We now factor by grouping

$$2x(3x + 5) - 1(3x - 5)$$

$$(3x + 5)(2x - 1)$$

Thus $6x^2 + 7x - 5 = (3x + 5)(3x - 1)$. □

Example 2. Factor $2x^2 + 7x - 4$.

Proof. Multiply the leading coefficient and the last term:

$$2(-4) = -8$$

We need to find two numbers that multiply to -8 and add to 7 (the middle term). These two numbers are 8 and -1. Replace the middle term with these two numbers, then factor by grouping:

$$2x^2 + (8 - 1)x - 4$$

$$2x^2 + 8x - x - 4$$

$$2x(x + 4) - 1(x - 4)$$

$$(2x - 1)(x + 4)$$

Thus $2x^2 + 7x - 4 = (2x - 1)(x + 4)$. □

Example 3. Factor $-4t^2 + 12t - 9$.

Proof. Multiply the leading coefficient and the last term:

$$(-4)(-9) = 36$$

We need to find two numbers that multiply to 36 and add to 12 (the middle term). These two numbers are 6 and 6. Replace the middle term with these two numbers, then factor by grouping:

$$-4t^2 + (6+6)t - 9$$

$$-4t^2 + 6t + 6t - 9$$

$$-2t(2t-3) + 3(2t-3)$$

$$(-2t+3)(2t-3)$$

We can also factor out the negative from the first factor:

$$-(2t-3)(2t-3)$$

$$-(2t-3)^2$$

Thus $-4t^2 + 12t - 9 = -(2t-3)^2$. □