Practice Problems II Solutions

① (a) $f(x) = x^2 - 8x + 13$

$= (x^2 - 8x + 16 - 16) + 13$

$= \boxed{(x-4)^2 - 3}$

f has $\boxed{\text{min}}$ since leading coeff is positive

min occurs at vertex $(4,-3)$, i.e. $\boxed{f(4) = -3}$

x-int: $(x-4)^2 - 3 = 0$

$(x-4)^2 = 3$

$x - 4 = \pm\sqrt{3}$

$x = 4 \pm \sqrt{3} \rightarrow \boxed{(4\pm\sqrt{3}, 0)}$

(b) $f(x) = -3x^2 + 12x - 6$

$= -3(x^2 - 4x + 4 - 4) - 6$

$= -3(x^2 - 4x + 4) + (-3)(-4) - 6$

$= \boxed{-3(x-2)^2 + 6}$

f has $\boxed{\text{max}}$ since leading coeff is negative

max occurs at vertex $(2,6)$, i.e. $\boxed{f(2) = 6}$

x-int: $-3(x-2)^2 + 6 = 0$

$-3(x-2)^2 = -6$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

$x = 2 \pm \sqrt{2} \rightarrow \boxed{(2\pm\sqrt{2}, 0)}$

② $h(t) = -16t^2 + 40t$

(a) $h(t) = 24 \Rightarrow -16t^2 + 40t = 24$ solve for t

$-16t^2 + 40t - 24 = 0$ divide through by -8

$2t^2 - 5t + 3 = 0$

$2t^2 - 2t - 3t + 3 = 0$

$2t(t-1) - 3(t-1) = 0$

$(2t-3)(t-1) = 0 \longrightarrow$ $\boxed{\text{at } t = 3/2 \text{ seconds and } t = 1 \text{ second}}$

(b) $h(t) = 48 \Rightarrow -16t^2 + 40t = 48$

$-16t^2 + 40t - 48 = 0$

$2t^2 - 5t + 6 = 0$ doesn't factor nicely, use quadratic formula

$t = \dfrac{5 \pm \sqrt{25 - 4(2)(6)}}{2(2)} = \dfrac{5 \pm \sqrt{-23}}{4}$ not a real #

$\boxed{\text{The object does not reach 48ft}}$

(c) What is the greatest height reached by the ball? Need to find maximum.

one way: complete the square:

$h(t) = -16t^2 + 40t$

$= -16\left(t^2 - \frac{40}{16}t\right)$

$= -16\left(t^2 - \frac{5}{2}t + \frac{25}{16} - \frac{25}{16}\right)$

$= -16\left(t^2 - \frac{5}{2}t + \frac{25}{16}\right) + (-16)\left(\frac{-25}{16}\right)$

$= -16\left(t - \frac{5}{4}\right)^2 + 25$

vertex: $\left(\frac{5}{4}, 25\right)$

$\boxed{\text{max height is 25 feet}}$

another way: $t = \dfrac{-b}{2a} = \dfrac{-40}{2(-16)} = \dfrac{40}{32} = \dfrac{5}{4}$

$h\left(\frac{5}{4}\right) = -16\left(\frac{5}{4}\right)^2 + 40\left(\frac{5}{4}\right)$

$= -25 + 50$

$= 25$

② (d) max height occurs at $\boxed{t = \frac{5}{4} \text{ seconds}}$ (see work in part (c))

(e) ground means $h(t) = 0 \Rightarrow$ $-16t^2 + 40t = 0$

$-8t(2t - 5) = 0$

$t = 0$ or $\boxed{t = 5/2 \text{ sec}}$

③ (a) $f(x) = x^3 + x^2 - x - 1$ factor by grouping

$= x^2(x+1) - 1(x+1)$

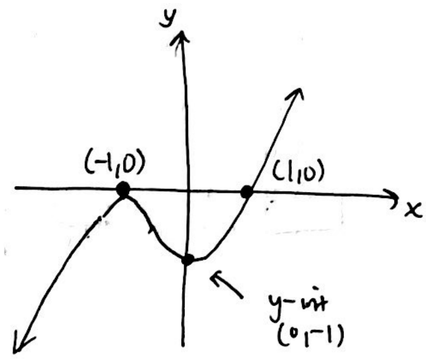
$= (x^2 - 1)(x+1)$

$= (x-1)(x+1)(x+1)$

$= (x-1)(x+1)^2$

zeros:	$x = 1$	$x = -1$
	mult 1	mult 2

cross x-axis — touch x-axis



has no real zeros

(b) $f(x) = \frac{1}{8}(2x^4 + 3x^3 - 16x - 24)^2 = \frac{1}{8}(x-2)^2(x^2+2x+4)^2(2x+3)^2$

factor the inside by grouping

$2x^4 + 3x^3 - 16x - 24$

$x^3(2x+3) - 8(2x+3)$

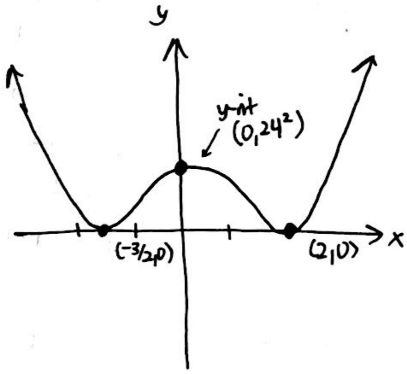
$(x^3 - 8)(2x+3)$

difference of cubes $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(x-2)(x^2+2x+4)(2x+3)$

zeros:	$x = 2$	$x = -3/2$
	mult 2	mult 2

touch x-axis — touch x-axis



(c) $f(x) = x^4 - 3x^2 - 4$

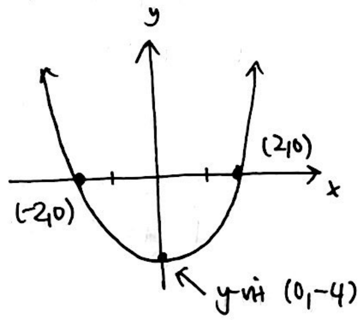
$= (x^2 - 4)(x^2 + 1)$

$= (x-2)(x+2)(x^2+1)$

no real zeros

zeros:	$x = 2$,	$x = -2$
	mult 1	mult 1

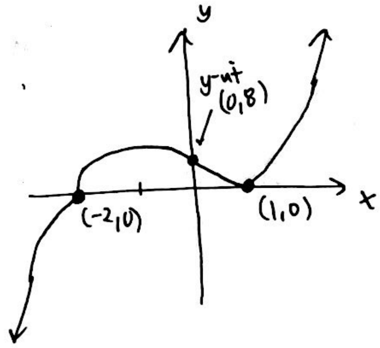
cross x-axis — cross x-axis



(d) $f(x) = (x-1)^2(x+2)^3$

zeros:	$x = 1$	$x = -2$
	mult 2	mult 3

touch x-axis — cross x-axis



4 (a) $f(x)=\dfrac{x^2-2x+1}{x^3-3x^2} = \dfrac{(x-1)^2}{x^2(x-3)}$

V.A. : $x^2(x-3)=0$

$x=0$ mult 2 $\quad x=3$ mult 1

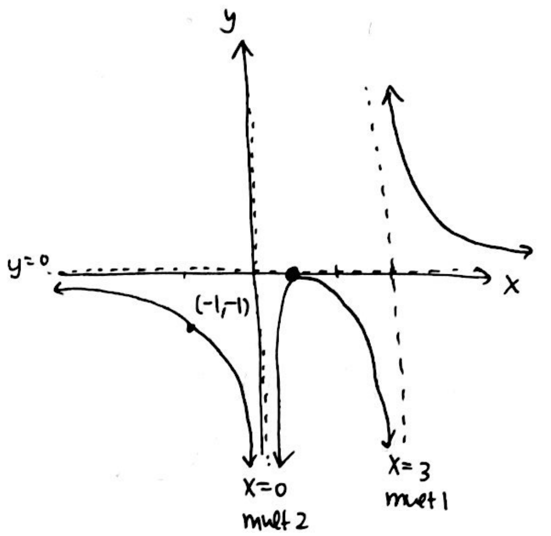
H.A. : $y=0$ since deg top < deg bottom

slant : no slant asymptote

x-int : $(x-1)^2=0 \Rightarrow x=1$ mult $2 \Rightarrow (1,0)$ touch

y-int : $\dfrac{(0-1)^2}{0^2(0-3)}$ undefined, no y-int

extra points : $f(-1)=\dfrac{(-1-1)^2}{(-1)^2(-1-3)} = \dfrac{4}{1(-4)} = -1$



(b) $f(x)=\dfrac{3x-x^2}{2x-2} = \dfrac{-x^2+3x}{2x-2} = \dfrac{-x(x-3)}{2x-2}$

V.A. : $2x-2=0$

$x=1$ mult 1

H.A. : none, deg top > deg bottom

slant :

$$\begin{array}{r} -\frac{x}{2}+1 \\ 2x-2\,\overline{)\,-\cancel{x^2}+3x} \\ -(-\cancel{x^2}+x) \\ \hline 2x-1 \\ -(2x-2) \\ \hline 1 \end{array}$$

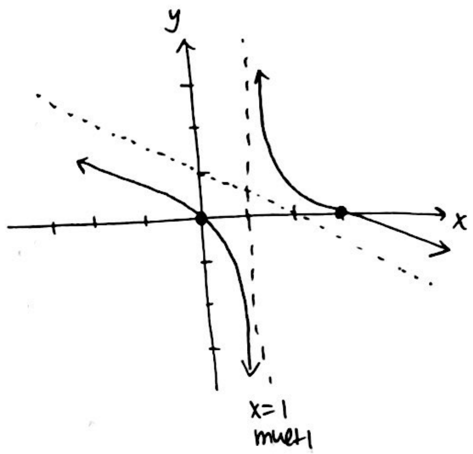
slant $\underline{y=-\frac{x}{2}+1}$

x-int : $-x(x-3)=0$

$x=0$ mult 1 $\quad x=3$ mult 1

$(0,0)$, $(3,0)$ cross

y-int : $\dfrac{-0(0-3)}{2(0)-2}=0$. $\quad (0,0)$



(c) $f(x)=\dfrac{2x(x+2)}{(x-1)(x-4)}$

V.A. : $(x-1)(x-4)=0 \quad x=1$ mult 1 $\quad x=4$ mult 1

H.A. : deg top = deg bottom $\quad y=\frac{2}{1}=2$

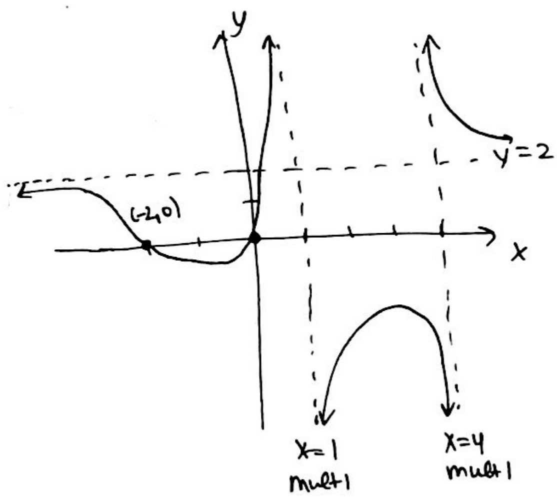
slant : no slant asymptote

x-int : $2x(x+2)=0$

$x=0$ mult 1 $\quad x=-2$ mult 1

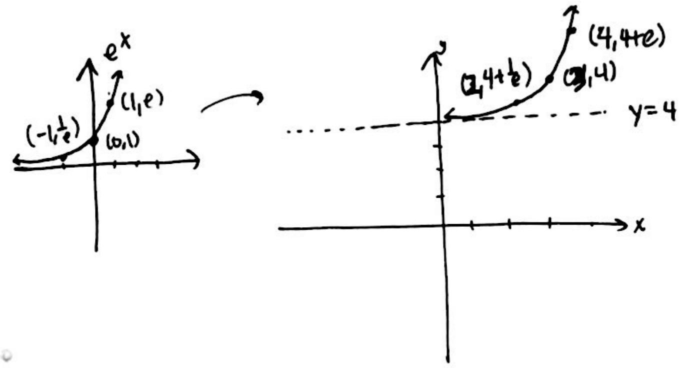
$(0,0)$, $(-2,0)$ cross

y-int : $\dfrac{2(0)(0+2)}{(0-1)(0-4)}=0 \quad (0,0)$



⑤ (a) $y = e^{x-3} + 4$

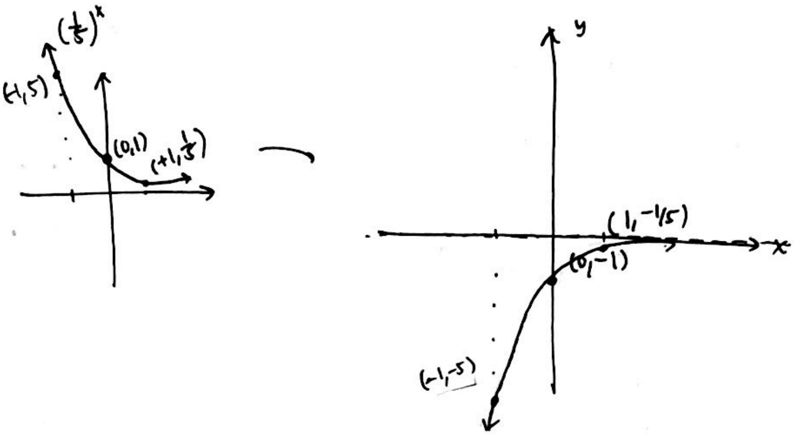
right 3, up 4



← this is the solution

(b) $f(x) = -\left(\frac{1}{5}\right)^x$

vertical reflection

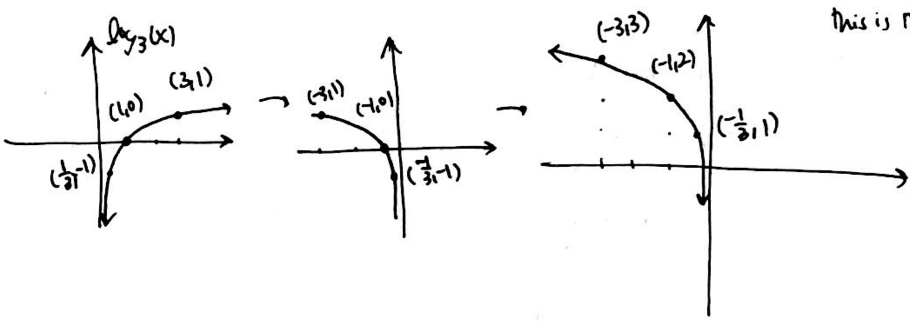


This is the solution

(c) $f(x) = \log_3(-x) + 2$

horizontal reflection

up 2

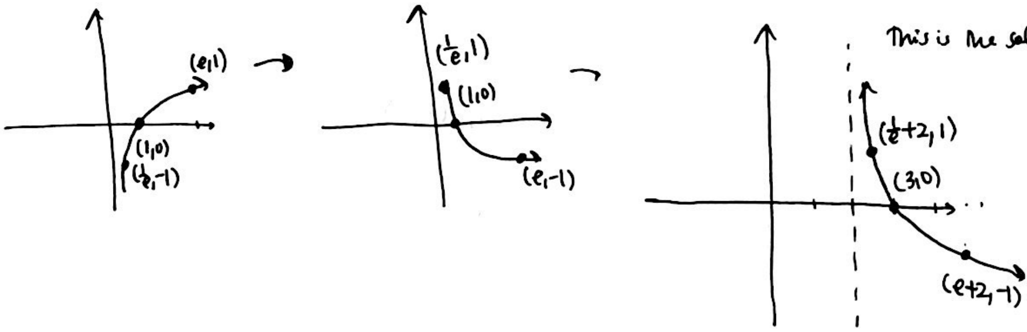


this is the solution

(d) $f(x) = -\ln(x-2)$

vertical reflection

right 2



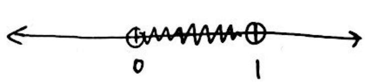
This is the solution

⑥ $\ln(x - x^2)$ Need $x - x^2 > 0$

$x(1-x) > 0$

$x(1-x) = 0$

$x = 0, \quad x = 1$



Domain: $\boxed{(0,1)}$

or $\{x : 0 < x < 1\}$

test points $x = -\frac{1}{2}$: $-\frac{1}{2}(1 + \frac{1}{2}) < 0$ ✗

$x = \frac{1}{2}$ $\frac{1}{2}(1 - \frac{1}{2}) > 0$ ✓

$x = 2$ $2(1-2) < 0$ ✗

⑦ (a) $\ln\left(\frac{x^3\sqrt{x-1}}{3x+4}\right) = \ln(x^3\sqrt{x-1}) - \ln(3x+4) = \ln x^3 + \ln(x-1)^{1/2} - \ln(3x+4)$

$= \boxed{3\ln x + \frac{1}{2}\ln(x-1) - \ln(3x+4)}$

(b) $\log\sqrt{x\sqrt{y\sqrt{z}}} = \frac{1}{2}\log x\sqrt{y\sqrt{z}} = \frac{1}{2}\left[\log x + \log(y\sqrt{z})^{1/2}\right]$

$= \frac{1}{2}\left[\log x + \frac{1}{2}\log y\sqrt{z}\right] = \frac{1}{2}\left[\log x + \frac{1}{2}(\log y + \log\sqrt{z})\right]$

$= \boxed{\frac{1}{2}\left[\log x + \frac{1}{2}(\log y + \frac{1}{2}\log z)\right]}$

⑧ $\underbrace{\frac{1}{3}\log(2x+1)} + \frac{1}{2}\left[\log(x-4) - \underbrace{3\log(x^4-x^2-1)}\right]$

$= \log(2x+1)^{1/3} + \frac{1}{2}\left[\underbrace{\log(x-4) - \log(x^4-x^2-1)^3}\right]$

$= \log(2x+1)^{1/3} + \underbrace{\frac{1}{2}}\log\left(\frac{x-4}{(x^4-x^2-1)^3}\right)$

$= \log(2x+1)^{1/3} + \log\left(\frac{x-4}{(x^4-x^2-1)^3}\right)^{1/2}$

$= \boxed{\log\left[(2x+1)^{1/3}\left(\frac{x-4}{(x^4-x^2-1)^3}\right)^{1/2}\right]}$

⑨ (a) $\ln(\ln e^{e^{200}}) = \ln(e^{200}\underbrace{\ln e}_{1}) = \ln(e^{200}) = 200\ln e = 200$

(b) $\underbrace{\log_2 6 - \log_2 15} + \log_2 20 = \log_2\frac{6}{15} + \log_2 20 = \log_2\left(\frac{6}{15}\cdot 20\right) = \log_2\left(\frac{120}{15}\right) = \log_2 8$

$= \log_2 2^3 = 3$

⑩ (a) $\log_5 x = 2 \Rightarrow 5^2 = x \Rightarrow x = 25$

(b) $\log_x 25 = 2 \Rightarrow x^2 = 25 \Rightarrow x = 5$

(c) $e^{4-x} = 2$

$\ln(e^{4-x}) = \ln 2$

$4 - x = \ln 2$

$\boxed{x = 4 - \ln 2}$

(d) $3^{x+2} = 7$

$\log_3 3^{x+2} = \log_3 7$

$x + 2 = \log_3 7$

$\boxed{x = \log_3 7 - 2}$

(e) $\log_2(x+2) = 5$

$2^{\log_2(x+2)} = 2^5$

$x + 2 = 32$

$x = 30$

(f) $4 + 3\log(2x) = 16$

$3\log(2x) = 12$

$\log(2x) = 4$

$10^{\log(2x)} = 10^4$

$2x = 10{,}000$

$\boxed{x = 5{,}000}$

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(10) (g) $x^2 2^x - 2^x = 0$

$2^x(x^2-1)=0$

$2^x(x-1)(x+1)=0$

$2^x \neq 0$ $\boxed{x=1 \quad x=-1}$

(h) $e^{2x} - 3e^x + 2 = 0$

$(e^x)^2 - 3e^x + 2 = 0$

take $u = e^x$

$u^2 - 3u + 2 = 0$

$(u-2)(u-1) = 0$

$u=2$ or $u=1$

$e^x = 2$ or $e^x = 1$

$\boxed{x = \ln 2 \text{ or } x = \ln 1 = 0}$

(i) $2\log x = \log 2 + \log(3x-4)$

$\log(x^2) = \log(2(3x-4))$

$10^{\log(x^2)} = 10^{\log(2(3x-4))}$

$x^2 = 2(3x-4)$

$x^2 = 6x-8$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x=4$ or $x=2$

check: if $x=2$ $\log(3x-4) = \log(-2)$ undefined

so $x=4$

(j) $2^{2/\log_5 x} = \frac{1}{16}$

$\log_2\left(2^{2/\log_5 x}\right) = \log_2 \frac{1}{16}$

$\frac{2}{\log_5 x} = -4$

$\log_5 x = -\frac{1}{2}$

$x = 5^{-1/2}$

$x = \frac{1}{\sqrt{5}}$

(k) $2^{3x+1} = 3^{x-2}$

doesn't matter the base, just pick one and stick with it

$\log_2 2^{3x+1} = \log_2 3^{x-2}$

$3x+1 = (x-2)\log_2 3$

$3x+1 = x\log_2 3 - 2\log_2 3$

$3x - x\log_2 3 = -1 - 2\log_2 3$

$x(3-\log_2 3) = -1 - 2\log_2 3$

$$\boxed{x = \frac{-1-2\log_2 3}{3-\log_2 3}}$$

* (11) use $\boxed{P(t) = P_0\left(1+\frac{r}{n}\right)^{nt}}$ P_0 = initial amount, r = rate, n = # times per year

(11) (a) $P_0 = 1000$

$t = 3$ yrs

$r = 0.02$

annually means $n=1$: $P(3) = 1000\left(1+\frac{0.02}{1}\right)^{1(3)} = 1000(1.02)^3 = \boxed{\$1061.21}$

quarterly means $n=4$: $P(3) = 1000\left(1+\frac{0.02}{4}\right)^{4(3)} = 1000(1.005)^{12} = \boxed{\$1061.68}$

monthly means $n=12$: $P(3) = 1000\left(1+\frac{0.02}{12}\right)^{12(3)} = 1000(1.00166\ldots)^{36} = \boxed{\$1061.78}$

weekly means $n=52$: $P(3) = 1000\left(1+\frac{0.02}{52}\right)^{52(3)} = 1000(1.0003846\ldots) = \boxed{\$1061.82}$

continuously use $P(t) = P_0 e^{rt} \Rightarrow$ $P(3) = 1000\, e^{0.02(3)} = \boxed{\$1061.84}$ ← continuously makes the most money

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11 (b) $P_0 = 1000$, $r = 0.02$, Find t

to double, need $2000 = 1000\,e^{0.02t}$

$$2 = e^{0.02t}$$

$$\ln 2 = \ln e^{0.02t}$$

$$\ln 2 = 0.02t \Rightarrow t = \frac{\ln 2}{0.02} \approx \boxed{34.7 \text{ years}}$$

(c) $P_0 = 1000$, $t = 10$ yrs, Find r

double means $2000 = 1000\,e^{r(10)}$

$$2 = e^{10r}$$

$$\ln 2 = 10r \Rightarrow r = \frac{\ln 2}{10} \approx 0.069 = 6.9\%$$

12 (a) $P = P_0 e^{rt}$, $P_0 = 300$ mg

need to find r. Half life means in 140 days, we should have half the amount

$$150 = 300\,e^{r(140)}$$

$$\frac{1}{2} = e^{140r}$$

$$\ln\left(\tfrac{1}{2}\right) = 140r \Rightarrow r = \frac{\ln\left(\frac{1}{2}\right)}{140}$$

* Note: this is a negative #, we expect negative rate with decay

Therefore $\boxed{P(t) = 300\,e^{\ln(1/2)t/140}}$

(b) one year means $t = 365$ days

$$P(365) = \boxed{300\,e^{\ln(1/2)\cdot 365/140}}$$

(c) Set $P(t) = 200$ and solve for t:

$$200 = 300\,e^{\ln(1/2)t/140}$$

$$\frac{2}{3} = e^{\ln(1/2)t/140}$$

$$\ln\left(\tfrac{2}{3}\right) = \frac{\ln\left(\frac{1}{2}\right)t}{140} \Rightarrow \boxed{t = \frac{140\ln(2/3)}{\ln(1/2)}} \text{ days}$$

13 (a) $P_0 = 8600$, Find r

$P(t) = 8600\,e^{rt}$ when $t = 1$, $P(t) = 10{,}000$

* typo, should be after 1 hour bacteria is 10,000

$$10{,}000 = 8600\,e^{r\cdot 1}$$

$$\frac{10{,}000}{8600} = e^{r} \Rightarrow r = \ln\left(\frac{100}{86}\right) = \ln\left(\frac{50}{43}\right)$$

Therefore $\boxed{P(t) = 8600\,e^{\ln(\frac{50}{43})t}}$

(b) $\boxed{P(2) = 8600\,e^{\ln(\frac{50}{43})\cdot 2}}$

(c) Double means twice 8600 $\Rightarrow$

$$2(8600) = 8600\,e^{\ln(\frac{50}{43})t}$$

$$2 = e^{\ln(50/43)t} \Rightarrow \ln 2 = \ln\left(\frac{50}{43}\right)t$$

$$\Rightarrow \boxed{t = \frac{\ln(2)}{\ln(50/43)}} \text{ hours}$$

* optional

(14) (a) $T_0 = 200^\circ F$ $\quad T(t) = 70^\circ + (200^\circ - 70^\circ)e^{rt}$ (we expect r to be negative)

$T_S = 70^\circ$ $\quad = 70 + 130e^{rt}$

when $t = 10$ min, $T(t) = 150^\circ$

$150 = 70 + 130e^{r(10)}$

$80 = 130e^{r(10)}$

$\frac{8}{13} = e^{r(10)}$

$\ln\left(\frac{8}{13}\right) = r(10) \longrightarrow r = \frac{\ln(8/13)}{10}$ (this is negative ✓)

Therefore $\boxed{T(t) = 70 + 130e^{\ln(8/13)t/10}}$

(b) $t = 15$, $\boxed{T(15) = 70 + 130e^{\ln(8/13)15/10}}$

(c) $100 = 70 + 130e^{\ln(8/13)t/10}$

$30 = 130e^{\ln(8/13)t/10}$

$\frac{3}{13} = e^{\ln(8/13)t/10}$

$\ln\left(\frac{3}{13}\right) = \frac{\ln(8/13)t}{10} \longrightarrow \boxed{t = \frac{10\ln(3/13)}{\ln(8/13)}}$ minutes