1. Simplify the following expressions:

   (a) $1 + 1 + 1 + 1 + 1 \times 0$

   Solution. Multiplication first: $1 + 1 + 1 + 1 + 0$
   Then add. The solution is 4. □

   (b) $7 + 7 \div 7 + 7 \times 7 - 7$

   Solution. Division first: $7 + 1 + 7 \times 7 - 7$
   Then multiplication: $7 + 1 + 49 - 7$
   Then perform addition and subtraction. The solution is 50. □

   (c) $6 \div 2(1 + 2)$

   Solution. Simplify inside the parentheses first: $6 \div 2(3)$
   Then division: $3(3)$
   Then multiply. The solution is 9. □

   (d) $9 - 3 \times \frac{1}{3} + 1$

   Solution. Multiplication first: $9 - 1 + 1$
   Then perform addition and subtraction. The solution is 9. □

2. Write the following expressions as one fraction (simplify as much as possible):

   (a) $\dfrac{3}{10} + \dfrac{4}{15}$

   Solution. Get a common denominator. The least common denominator is 30:

   $$\frac{3}{3} \cdot \frac{3}{10} + \frac{4}{15} \cdot \frac{2}{2} = \frac{9}{30} + \frac{8}{30} = \frac{17}{30}$$

   □

   (b) $\dfrac{3}{10} \cdot \dfrac{4}{15}$

   Solution. Multiply the numerators and denominators:

   $$\frac{3}{10} \cdot \frac{4}{15} = \frac{3 \cdot 4}{10 \cdot 15} = \frac{12}{150} = \frac{2}{25}$$

   □

Written by Victoria Kala, vtkala@math.ucla.edu

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(c) $\dfrac{\frac{3}{10}}{\frac{4}{15}}$

Solution. Multiply by the recriprocal of the denominator (i.e. flip the bottom fraction and multiply):

$$\frac{3}{10} \cdot \frac{15}{4} = \frac{3 \cdot 15}{10 \cdot 4} = \frac{45}{40} = \frac{9}{8}$$

□

3. Simplify $8(2x + 5) - 7(x - 9)$

Proof. Distribute and combine like terms (watch out for signs!):

$$16x + 40 - 7x + 63 = 9x + 103$$

□

4. Multiply $(2x + 5)^2$

Proof. Rewrite as a product and distribute:

$$(2x+5)^2 = (2x+5)(2x+5) = 2x(2x+5)+5(2x+5) = 4x^2+10x+10x+25 = 4x^2+20x+25$$

□

5. Factor:

(a) $x^2 - 4x + 4$

Proof. Find two factors of $+4$ (last term) whose sum is -4 (coefficient of x term). The factors are -2 and -2:

$$x^2 - 4x + 4 = (x - 2)(x - 2) = (x - 2)^2$$

□

(b) $9x^2 - 16$

Solution. Use the difference of squares formula $a^2 - b^2 = (a - b)(a + b)$:

$$9x^2 - 16 = (3x - 4)(3x + 4)$$

□

(c) $2x^2 - 5x - 3$

Solution. Find two factors of $2(-3)$ (the coefficient of x^2 term multiplied by the last term) that add to -5 (coefficient of x term). The factors are -6 and $+1$.
Split up the middle term in terms of the two factors:

$$2x^2 - 5x - 3$$

$$2x^2 + (-6+1)x - 3$$

$$2x^2 - 6x + 1x - 3$$

Factor the first two terms and the last two terms:

$$2x(x-3) + 1(x-3)$$

These both have an $(x-3)$ in common, factor that out:

$$(x-3)(2x+1)$$

(Note: You may have another way of factoring this to get the correct solution and that is okay!) □

(d) $x^3 + 8$

Solution. Use the sum of cubes formula $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$:

$$x^3 + 8 = (x+2)(x^2 - 2x + 4)$$

□

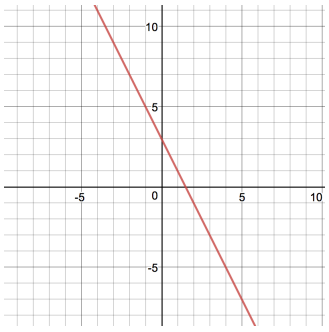
6. Consider the line $y = -2x + 3$.

(a) What is the slope of the line? What is the y-intercept of the line?

Solution. Recall that a the line $y = mx+b$ has slope m and y-intercept b. For $y = -2x+3$, the slope is -2 and the y-intercept is 3. □

(b) Sketch the graph of the line.

Solution.



□

Challenge (Optional)

7. Write the following expressions as one fraction (simplify as much as possible):

   (a) $\dfrac{1}{x} - \dfrac{2}{x(x+1)}$

   Solution. Get a common denominator. The least common denominator is $x(x+1)$:

   $$\frac{x+1}{x+1} \cdot \frac{1}{x} - \frac{2}{x(x+1)} = \frac{x+1-2}{x(x+1)} = \frac{x-1}{x(x+1)}$$

   □

   (b) $\dfrac{x-4}{x^2-4} \div \dfrac{x^2-3x-4}{x^2+5x+6}$

   Solution. Factor each of the numerators and denominators:

   $$\frac{x-4}{(x-2)(x+2)} \div \frac{(x-4)(x+1)}{(x+3)(x+2)}$$

   Flip the second fraction and multiply:

   $$\frac{x-4}{(x-2)(x+2)} \cdot \frac{(x+3)(x+2)}{(x-4)(x+1)} = \frac{(x-4)(x+3)(x+2)}{(x-2)(x+2)(x-4)(x+1)} = \frac{x+3}{(x-2)(x+1)}$$

   □