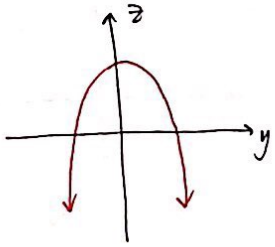
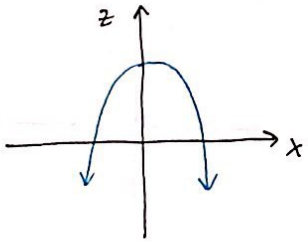
(1) $x^2+y^2+z=3$ "elliptic paraboloid"

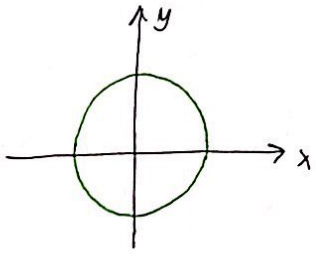
x trace: $y^2+z=3$, $x=0$, $z=3-y^2$ parabola



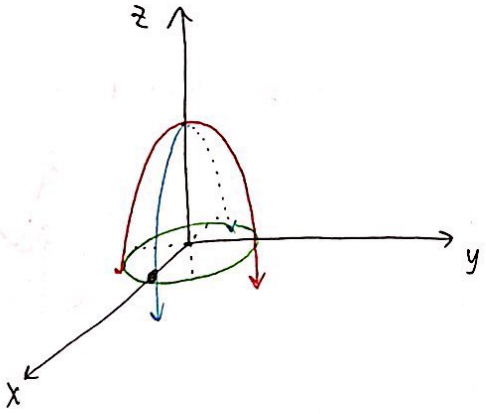
y trace: $x^2+z=3$, $y=0$, $z=3-x^2$ parabola



z trace: $x^2+y^2=3$, $z=0$, circle

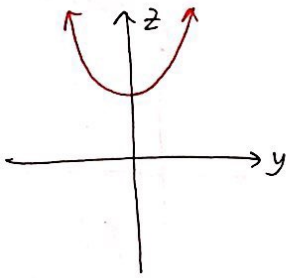


In 3D:

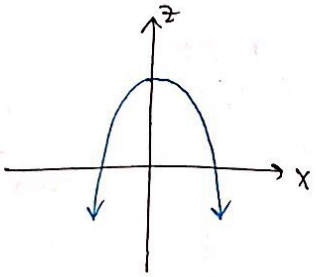


$x^2-y^2+z=3$ "hyperbolic paraboloid"

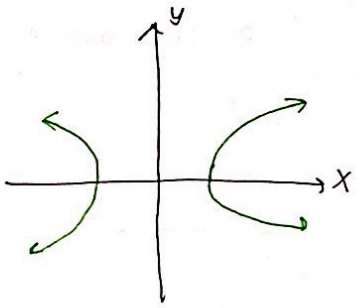
x trace: $-y^2+z=3$, $x=0$, $z=3+y^2$ parabola



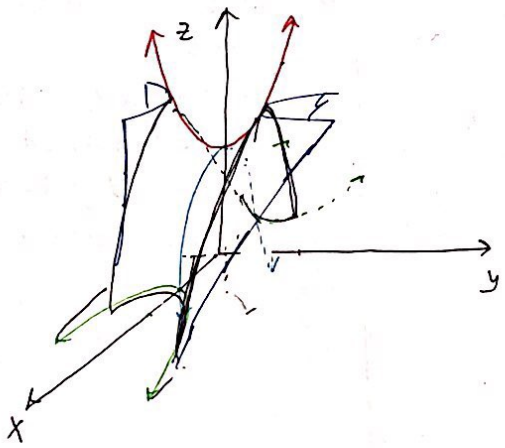
y trace: $x^2+z=3$, $y=0$, $z=3-x^2$ parabola



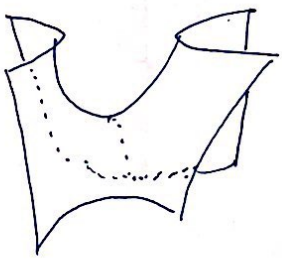
z trace: $x^2-y^2=3$, $z=0$, hyperbola



In 3D:

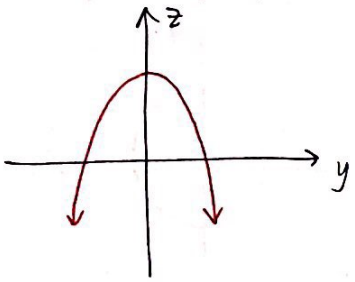


difficult to draw in 3D

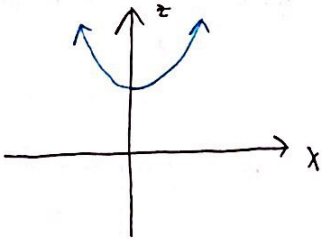


$-x^2+y^2+z=3$ · "hyperbolic paraboloid"

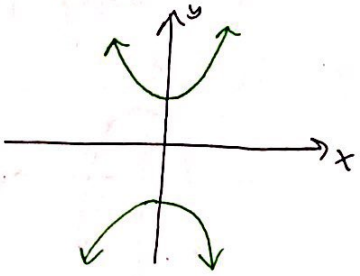
x trace: $y^2+z=3$, $x=0$, $z=3-y^2$ parabola



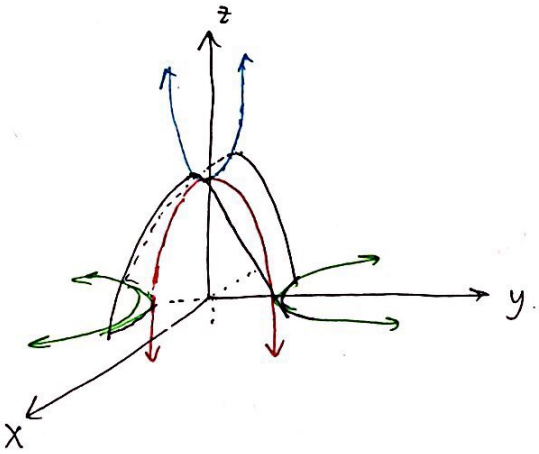
y trace: $-x^2+z=3$, $y=0$, $z=3+x^2$ parabola



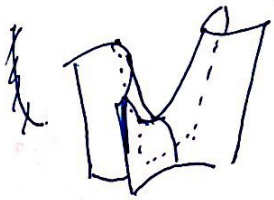
z trace: $-x^2+y^2=3$, $z=0$, $y^2-x^2=3$ hyperbola



In 3D:

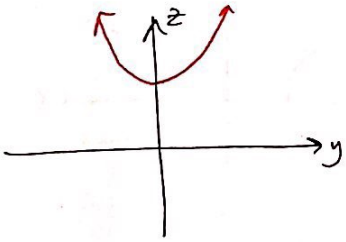


difficult to draw in 3D

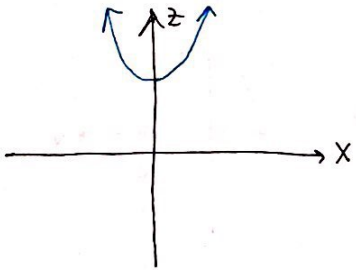


$-x^2-y^2+z=3$ "elliptic paraboloid"

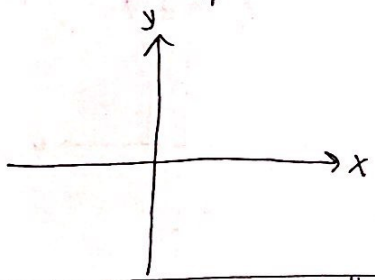
x trace: $-y^2+z=3$, $x=0$, $z=3+y^2$ parabola



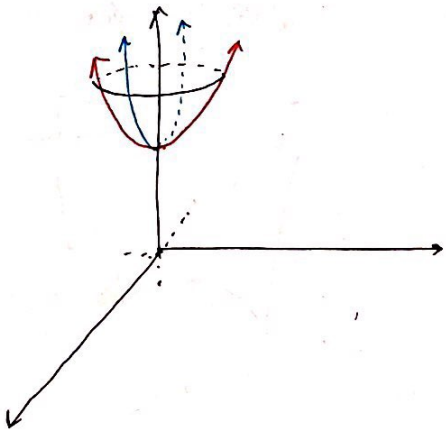
y trace: $-x^2+z=3$, $y=0$, $z=3+x^2$ parabola



z trace: $-x^2-y^2=3$, $z=0$, $x^2+y^2=-3$ impossible



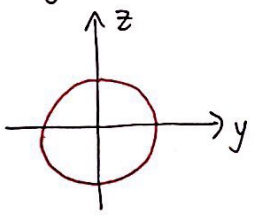
In 3D:



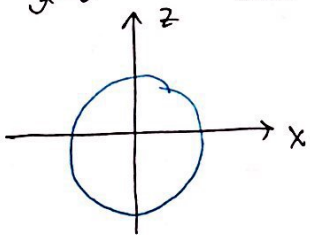
* (c) · all these surfaces have parabolas as traces
  - If x^2 and y^2 have the same sign then elliptic paraboloid
  - If x^2 and y^2 have opposite signs then hyperbolic paraboloid
  - z has no exponent in the equation

(2) $x^2+y^2+z^2=1$ "sphere"

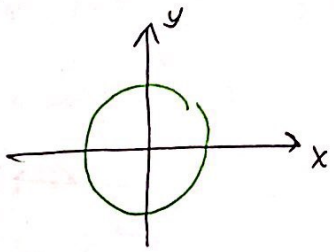
x trace: $y^2+z^2=1$ circle
$x=0$



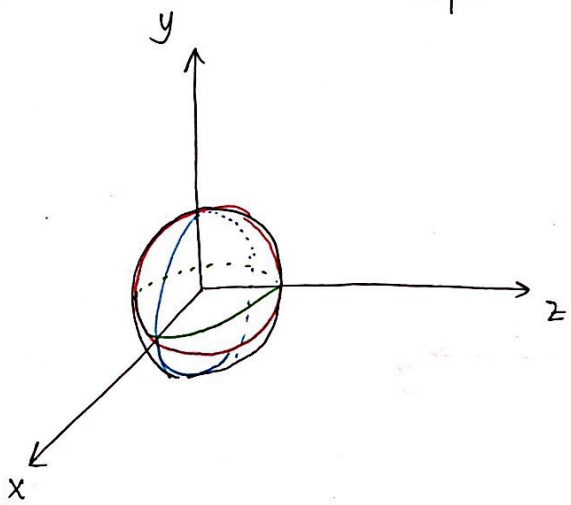
y trace $x^2+z^2=1$ circle
$y=0$



z trace $x^2+y^2=1$ circle
$z=0$



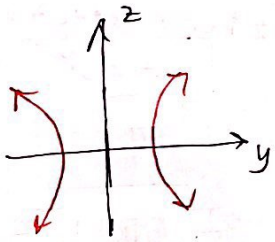
In 3D:



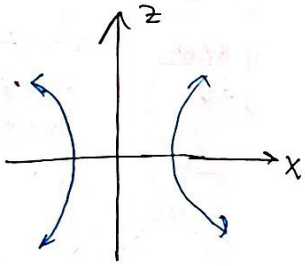
notice one negative ⇒ one sheet
"hyperboloid of one sheet"

$x^2+y^2-z^2=1$

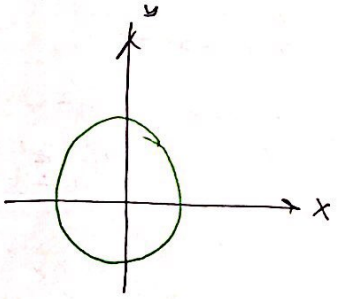
x trace
$x=0$: $y^2-z^2=1$ hyperbola



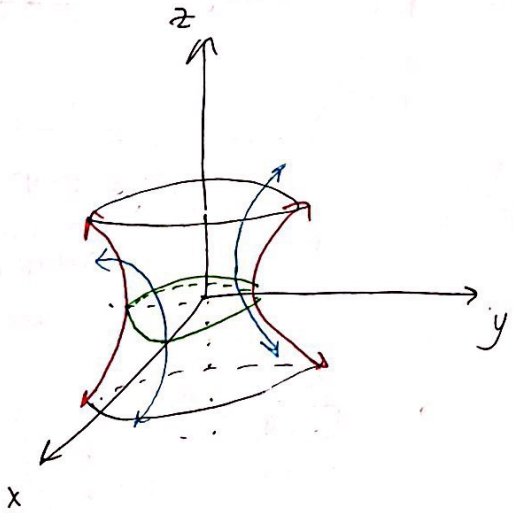
y trace
$y=0$ $x^2-z^2=1$ hyperbola



z trace
$z=0$ $x^2+y^2=1$



In 3D:

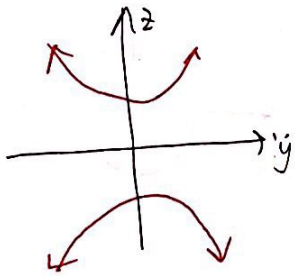


(2) continued

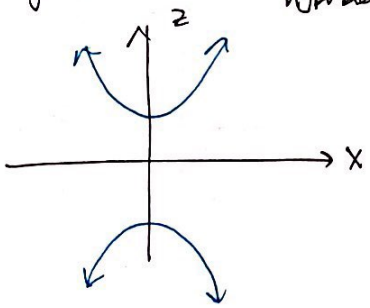
$x^2+y^2-z^2=-1$ $\quad (z^2-x^2-y^2=1)$ "hyperboloid of two sheets"

notice 2 negatives in equation ⇒ two sheets

x trace: $x=0$, $y^2-z^2=-1$, $z^2-y^2=1$ hyperbola

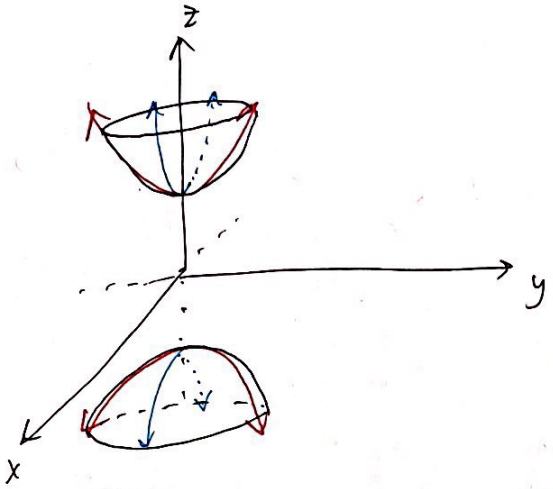


y trace: $y=0$, $x^2-z^2=-1$, $z^2-x^2=1$ hyperbola



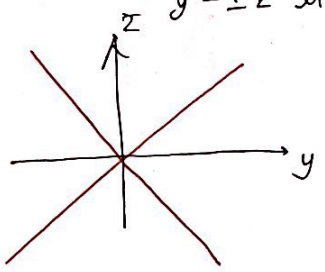
z trace: $z=0$, $x^2+y^2=-1$ impossible

In 3D:

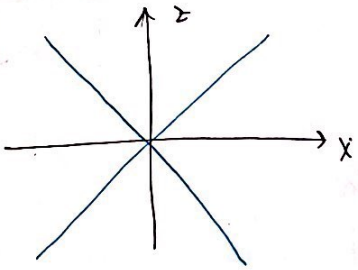


$x^2+y^2-z^2=0$ $\quad (z=\pm\sqrt{x^2+y^2})$ "cone"

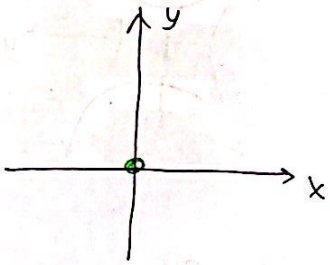
x trace: $x=0$, $y^2-z^2=0$, $y^2=z^2$, $y=\pm z$ lines



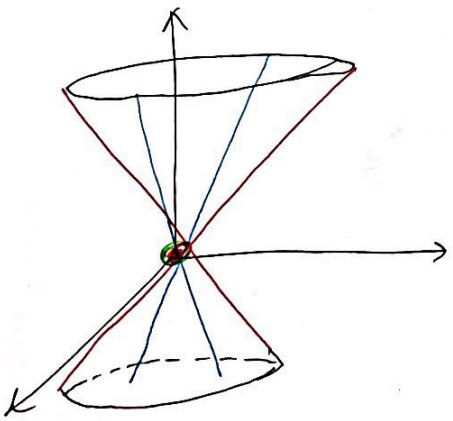
y trace: $y=0$, $x^2-z^2=0$, $x^2=z^2$, $x=\pm z$ lines



z trace: $x^2+y^2=0$, $x=0, y=0$ point



In 3D:



(c) $x^2-y^2+z^2=1$ hyperboloid of one sheet (because 1 negative in equation), opening in y-axis direction, e.g.