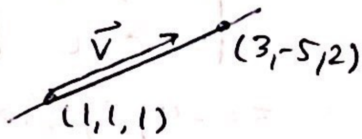
Quiz 1 Thursday

① $\vec{u} = \langle 2,3 \rangle \quad \|\vec{u}\| = \sqrt{2^2+3^2} = \sqrt{4+9} = 13$

$\frac{1}{13}\langle 2,3\rangle$ is a unit vector, and so is $\frac{-1}{13}\langle 2,3\rangle \Rightarrow \lambda = \pm\frac{1}{13}$

② $\vec{r}(t) = \vec{r}_0 + t\vec{v}$ There are multiple solution to this problem.

$\vec{r}_0 = \langle 1,1,1\rangle$

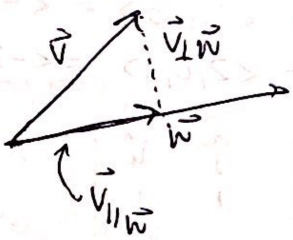


$\vec{v} = \langle 3,-5,2\rangle - \langle 1,1,1\rangle$
$= \langle 2,-6,1\rangle$

$$\Rightarrow \boxed{\begin{aligned}\vec{r}(t) &= \langle 1,1,1\rangle + t\langle 2,-6,1\rangle \\ &= \langle 2t+1, -6t+1, t+1\rangle\end{aligned}}$$

Quiz 1 Tuesday

① $\vec{v} = \langle 4,-1,5\rangle$
$\vec{w} = \langle 0,1,1\rangle$



$$\vec{v}_{\parallel\vec{w}} = \frac{\vec{v}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w} = \frac{\langle 4,-1,5\rangle\cdot\langle 0,1,1\rangle}{\langle 0,1,1\rangle\cdot\langle 0,1,1\rangle}\langle 0,1,1\rangle$$
$$= \frac{0-1+5}{1+1}\langle 0,1,1\rangle$$
$$= \langle 0,2,2\rangle$$

$\vec{v}_{\perp\vec{w}} = \vec{v} - \vec{v}_{\parallel\vec{w}} = \langle 4,-1,5\rangle - \langle 0,2,2\rangle = \langle 4,-3,3\rangle$

$$\boxed{\vec{v} = \underbrace{\langle 0,2,2\rangle}_{\text{prellel to } \vec{w}} + \underbrace{\langle 4,-3,3\rangle}_{\perp \text{ to } \vec{w}}}$$

② $(\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) \times (\hat{\jmath} - \hat{k}) \overset{\text{distribute}}{=} \hat{\imath}\times\hat{\jmath} - \hat{\imath}\times\hat{k} - 3\cancel{\hat{\jmath}\times\hat{\jmath}}^{\,0} + 3\hat{\jmath}\times\hat{k} + 2\hat{k}\times\hat{\jmath} - 2\cancel{\hat{k}\times\hat{k}}^{\,0}$

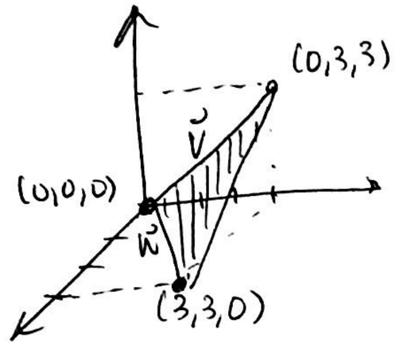
$= \hat{k} - (-\hat{\jmath}) + 3\hat{\imath} + 2(-\hat{\imath})$

$= \hat{\imath} + \hat{\jmath} + \hat{k} = \langle 1,1,1\rangle$

Quiz 1 Thursday

① same as ~~Q~~Quiz 1 Tuesday #1

②



$\vec{v} = \langle 0,3,3\rangle - \langle 0,0,0\rangle = \langle 0,3,3\rangle$

$\vec{w} = \langle 3,3,0\rangle - \langle 0,0,0\rangle = \langle 3,3,0\rangle$

Area is $\frac{\|\vec{v} \times \vec{w}\|}{2}$

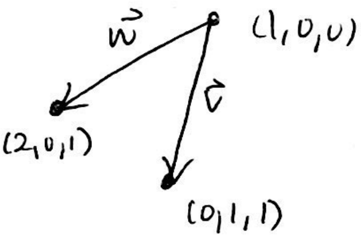
$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 3 \\ 3 & 3 & 0 \end{vmatrix} = \hat{i}\underset{0-9}{\begin{vmatrix} 3 & 3 \\ 3 & 0 \end{vmatrix}} - \hat{j}\underset{0-9}{\begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix}} + \hat{k}\underset{0-9}{\begin{vmatrix} 0 & 3 \\ 3 & 3 \end{vmatrix}}$$

$$= \langle -9, 9, -9\rangle$$

$$\Rightarrow \text{Area} = \frac{\sqrt{(-9)^2 + 9^2 + (-9)^2}}{2} = \frac{\sqrt{3(9)^2}}{2} = \frac{9\sqrt{3}}{2}$$

Quiz 2 Tuesday

① plane $\vec{n} \cdot \langle x - x_0, y - y_0, z - z_0\rangle = 0$ need normal vector



$\vec{v} = \langle 0,1,1\rangle - \langle 1,0,0\rangle = \langle -1,1,1\rangle$

$\vec{w} = \langle 2,0,1\rangle - \langle 1,0,0\rangle = \langle 1,0,1\rangle$

$$\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}\underset{1-0}{\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}} - \hat{j}\underset{-1-1}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}} + \hat{k}\underset{0-1}{\begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}}$$

$$= \langle 1, 2, -1\rangle$$

Use any given point for (x_0, y_0, z_0) $\Rightarrow$

$$\langle 1,2,-1\rangle \cdot \langle x-1, y-0, z-0\rangle = 0$$

$$x - 1 + 2y - z = 0$$

$$x + 2y - z = 1$$

② $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 - 5z^2 = 1$

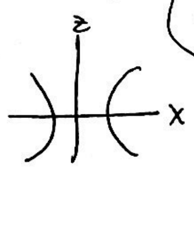
one negative

$\boxed{\text{hyperboloid of one sheet}}$

at $y=1$ $\left(\frac{x}{3}\right)^2 + \frac{1}{25} - 5z^2 = 1$

$\left(\frac{x}{3}\right)^2 - 5z^2 = \frac{24}{25}$ hyperbola that open toward x-axis

e.g.



Quiz 3 Thursday

① Same as Quiz 2 Tuesday #2

② $\vec{r}(t) = \langle 7, 12\cos t, 12\sin t\rangle$

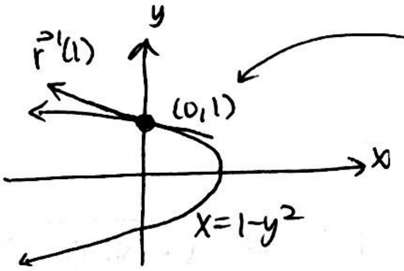
$x = 7$
$y = 12\cos t$
$z = 12\sin t$

$y^2 + z^2 = 12^2\cos^2 t + 12^2\sin^2 t$
$= 12^2(\cos^2 t + \sin^2 t)$
$= 144$

circle w/ radius 12 in plane $x = 7$

Quiz 3 Tuesday

① $\vec{r}(t) = \langle 1-t^2, t\rangle$



at $t=1$, $\vec{r}(1) = \langle 0, 1\rangle$

$x = 1-t^2$
$y = t$
$x = 1-y^2$

$\vec{r}'(t) = \langle -2t, 1\rangle$

$\vec{r}'(1) = \langle -2, 1\rangle$ i.e. left 2 up 1

② $\frac{d}{dt}\left((t^2, t^3, t) \times (e^{3t}, e^{2t}, e^t)\right) \underset{\text{product rule}}{=} \frac{d}{dt}(t^2, t^3, t) \times (e^{3t}, e^{2t}, e^t) + (t^2, t^3, t) \times \frac{d}{dt}(e^{3t}, e^{2t}, e^t)$

$= (2t, 3t^2, 1) \times (e^{3t}, e^{2t}, e^t) + (t^2, t^3, t) \times (3e^{3t}, 2e^{2t}, e^t)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2t & 3t^2 & 1 \\ e^{3t} & e^{2t} & e^t \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^2 & t^3 & t \\ 3e^{3t} & 2e^{2t} & e^t \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 3t^2 & 1 \\ e^{2t} & e^t \end{vmatrix} - \hat{j}\begin{vmatrix} 2t & 1 \\ e^{3t} & e^t \end{vmatrix} + \hat{k}\begin{vmatrix} 2t & 3t^2 \\ e^{3t} & e^{2t} \end{vmatrix} + \hat{i}\begin{vmatrix} t^3 & t \\ 2e^{2t} & e^t \end{vmatrix} - \hat{j}\begin{vmatrix} t^2 & t \\ 3e^{3t} & e^t \end{vmatrix} + \hat{k}\begin{vmatrix} t^2 & t^3 \\ 3e^{3t} & 2e^{2t} \end{vmatrix}$$

$= \langle 3t^2e^t - e^{2t}, -(2te^t - e^{3t}), 2te^{2t} - 3t^2e^{3t}\rangle + \langle t^3e^t - t2e^{2t}, -(t^2e^t - 3te^{3t}), 2t^2e^{2t} - 3t^3e^{3t}\rangle$

$= \underline{\langle 3t^2e^t - e^{2t} + t^3e^t - 2te^{2t}, -2te^t + e^{3t} - t^2e^t + 3te^{3t}, 2te^{2t} - 3t^2e^{3t} + 2t^2e^{2t} - 3t^3e^{3t}\rangle}$

Quiz 4 Thursday

① same as Quiz 3 Thursday #2

② $\vec{r}(t) = \langle \cos \pi t, \sin \pi t, t \rangle$

$\vec{r}'(t) = \langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle$ $\quad \|\vec{r}'(1)\| = \sqrt{\pi^2 \sin^2 \pi t + \pi^2 \cos^2 \pi t + 1}$

$= \sqrt{\pi^2 + 1}$

$\vec{T}(t) = \dfrac{\langle -\pi \sin \pi t, \pi \cos \pi t, 1 \rangle}{\sqrt{\pi^2+1}} \Rightarrow \quad \vec{T}(1) = \dfrac{\langle -\pi \sin \pi, \pi \cos \pi, 1 \rangle}{\sqrt{\pi^2+1}} = \dfrac{\langle 0, -\pi, 1 \rangle}{\sqrt{\pi^2+1}}$

Quiz 4 Tuesday

① $\vec{r}(\theta) = \langle R\cos\theta, R\sin\theta \rangle$

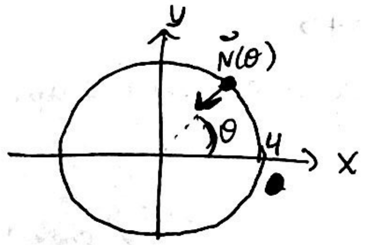
$\vec{r}'(\theta) = \langle -R\sin\theta, R\cos\theta \rangle \quad \|\vec{r}'(\theta)\| = \sqrt{R^2\sin^2\theta + R^2\cos^2\theta} = R$

$\vec{T}(\theta) = \dfrac{\langle -R\sin\theta, R\cos\theta \rangle}{R} = \langle -\sin\theta, \cos\theta \rangle$

$\boxed{\vec{N}(\theta) = \langle \cos\theta, \sin\theta \rangle}$

$\vec{N}\left(\frac{\pi}{4}\right) = \langle \cos\frac{\pi}{4}, \sin\left(\frac{\pi}{4}\right) \rangle$

$= \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$



② $\vec{r}(t) = \langle e^t, e^{-t}, t^2 \rangle$

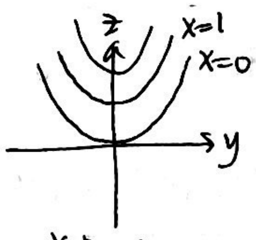
$\vec{r}'(t) = \langle e^t, -e^{-t}, 2t \rangle \quad \vec{T}(t) = \dfrac{\langle e^t, -e^{-t}, 2t \rangle}{\sqrt{e^{2t}+e^{-2t}+4t^2}} \quad \vec{T}(1) = \dfrac{\langle e, -e^{-1}, 2 \rangle}{\sqrt{e^2+e^{-2}+4}}$

$\|\vec{r}'(t)\| = \sqrt{e^{2t}+e^{-2t}+4t^2}$

Quiz 5 Thursday

① $f(x,y) = x^2+4y^2$

$z = x^2+4y^2$

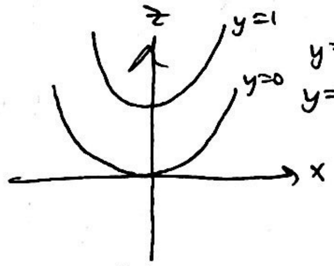


x trace

parabolas

$x=0 \quad z=4y^2$

$x=1 \quad z=1+4y^2$



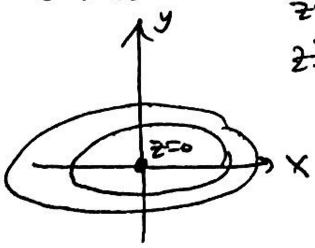
y trace

parabolas

$y=0 \quad z=x^2$

$y=1 \quad z=x^2+4$

z trace

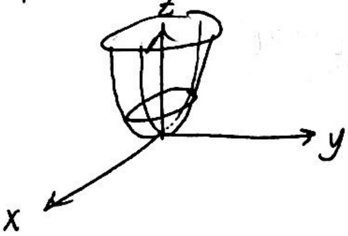


ellipses

$z=0 \quad x^2+4y^2$

$z=1 \quad x^2+4y^2$

In 3D: paraboloid



② $\lim_{(x,y)\to(2,5)} e^{f(x,y)^2 - g(x,y)} = e^{3^2-7} = e^2$

QUIZ 5 Tuesday

① same as Quiz 5 Thursday #1

② $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{\sqrt{x^2+y^2}}$ Try $x=0$: $\lim_{y\to 0} \frac{0-y^2}{\sqrt{0+y^2}} = \lim_{y\to 0} \frac{-y^2}{y} = \lim_{y\to 0} -y = 0.$

Try $y=0$ $\lim_{x\to 0} \frac{x^2-0}{\sqrt{x^2+0^2}} = \lim_{x\to 0} \frac{x^2}{x} = \lim_{x\to 0} x = 0.$

Try $y=mx$: $\lim_{x\to 0} \frac{x^2-m^2x^2}{\sqrt{x^2+m^2x^2}} = \lim_{x\to 0} \frac{(1-m^2)x^2}{\sqrt{1+m^2}\ x} = \lim_{x\to 0} \frac{(1-m^2)\ x}{\sqrt{1+m^2}} = 0$

Looks like the limit exists, but we need to prove by changing to polar coordinates!

$x=r\cos\theta$, $y=r\sin\theta$: $\lim_{r\to 0} \frac{r^2\cos^2\theta - r^2\sin^2\theta}{\sqrt{r^2\cos^2\theta + r^2\sin^2\theta}} = \lim_{r\to 0} \frac{r^2(\cos^2\theta-\sin^2\theta)}{r}$

each between 1 and -1

$= \lim_{r\to 0} r(\cos^2\theta - \sin^2\theta) = 0$ since squeeze: $-2r \le r(\cos^2\theta-\sin^2\theta) \le 2r$

$\lim_{r\to 0} -2r \le \lim_{r\to 0} r(\cos^2\theta-\sin^2\theta) \le \lim_{r\to 0} 2r$

Therefore limit exists and $=0$.

$0 \le$ " ≤ 0

QUIZ 6 Thursday

① $h(x,z) = e^{xz-x^2z^3}$ $h_z = e^{xz-x^2z^3} \frac{\partial}{\partial z}(xz-x^2z^3) = (x-3x^2z^2)e^{xz-x^2z^3}$

$h_z(3,0) = (3-0)e^{0-0} = 3$

② same as Quiz 5 Tuesday #2

Quiz 6 Tuesday

① same as Quiz 6 Thursday #1

② $x^2+y^2-z^2=6$ $\nabla f = \langle 2x, 2y, -2z \rangle$

Define $f = x^2+y^2-z^2$ $\nabla f(3,1,2) = \langle 6, 2, -4 \rangle$

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Quiz 7 Thursday

① Same as Quiz 6 Tuesday #2

② $f(x,y) = \cos(x^2+y)$ $\qquad f_x = -\sin(x^2+y)\cdot 2x$

$f_y = -\sin(x^2+y)\cdot 1$

$\nabla f = \langle -2x\sin(x^2+y), -\sin(x^2+y)\rangle$

Quiz 7 Tuesday

① same as Quiz 7 Thursday #2

② $f(x,y) = x^2+y^2-xy+x$ $\qquad f_x = 2x-y+1=0 \rightarrow y=2x+1$ plug into 2nd equation

$f_y = 2y-x=0 \rightarrow 2(2x+1)-x=0$

$4x+2-x=0$

$3x+2=0 \qquad x=-2/3$

$y = 2(\frac{-2}{3})+1 = 1-\frac{4}{3} = \frac{-1}{3}$

$(\frac{-2}{3}, \frac{-1}{3})$ is critical pt

$f_{xx} = 2 > 0 \qquad D = 2(2)-(-1)^2 = 4-1=3>0 \rightarrow$ local min

$f_{yy} = 2$

$f_{xy} = -1$

Quiz 8 Thursday

① Same as Quiz 7 Tuesday #2

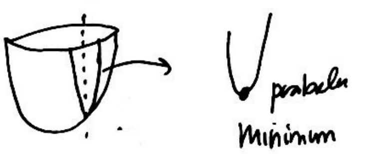
② $f(x,y) = x^2+y^2$ $\qquad \nabla f = \langle 2x, 2y\rangle$

$\underbrace{2x+3y}_{g} = 6 \qquad \nabla g = \langle 2,3\rangle$

$$\begin{cases} 2x = \lambda 2 \rightarrow \lambda = x \text{ plug into 2nd equation} \\ 2y = \lambda 3 \rightarrow 2y = 3x \rightarrow x = \frac{2}{3}y \text{ plug into 3rd equation} \\ 2x+3y=6 \rightarrow 2(\frac{2}{3}y)+3y=6 \end{cases}$$

$\frac{4}{3}y + 3y = 6$

$\frac{13}{3}y = 6 \qquad y = \frac{18}{13} \qquad x=2y$ so $x = \frac{2}{3}(\frac{18}{13}) = \frac{12}{13} \qquad (\frac{12}{13}, \frac{18}{13})$ critical pt



no maximum

$f(\frac{12}{13}, \frac{18}{13}) = (\frac{12}{13})^2 + (\frac{18}{13})^2 = \frac{144+324}{13^2} = \frac{468}{13^2} = \frac{36}{13}$ minimum value

Quiz 8 Tuesday

① $f(x,y) = y^2x - yx^2 + xy$

(a) $f_x = y^2-2xy+y = y(y-2x+1) = 0$ ✓

$f_y = 2yx - x^2 + x = x(2y-x+1) = 0$

Scanned with CamScanner

① (b) $y(y-2x+1)=0 \longrightarrow y=0$ or $y=2x-1$ plug both into 2nd equation

$x(2y-x+1)=0$

If $y=0$: $x(-x+1)=0 \Rightarrow x=0$ or $x=1$. $\boxed{(0,0),(1,0)}$

If $y=2x-1$: $x(4x-2-x+1)=0$

$x(3x-1)=0 \longrightarrow x=0$ or $x=\frac{1}{3}$ $\boxed{(0,-1),\left(\frac{1}{3},-\frac{1}{3}\right)}$

If $x=0$, $y=2(0)-1=-1$

$x=\frac{1}{3}$, $y=2\cdot\frac{1}{3}-1=-\frac{1}{3}$

(c) $f_{xx}=-2y$

$f_{yy}=2x$

$f_{xy}=2y-2x+1$

At $(0,0)$: $f_{xx}=0$, $f_{yy}=0$, $f_{xy}=1$ $\quad D=0-(1)^2=-1<0$ $\boxed{\text{saddle at } (0,0)}$

At $(1,0)$: $f_{xx}=0$, $f_{yy}=2$, $f_{xy}=-1$ $\quad D=0-(-1)^2<0$ $\boxed{\text{saddle at } (1,0)}$

At $(0,-1)$: $f_{xx}=2$, $f_{yy}=0$, $f_{xy}=-1$ $\quad D=0-(-1)^2<0$ $\boxed{\text{saddle at } (0,-1)}$

At $\left(\frac{1}{3},-\frac{1}{3}\right)$: $f_{xx}=\frac{2}{3}>0$ $\quad D=\left(\frac{2}{3}\right)^2-\left(\frac{-1}{3}\right)^2=\frac{4}{9}-\frac{1}{9}=\frac{3}{9}=\frac{1}{3}>0$ $\boxed{\text{local min at } \left(\frac{1}{3},-\frac{1}{3}\right)}$

$f_{yy}=\frac{2}{3}$

$f_{xy}=-\frac{2}{3}-\frac{2}{3}+1=-\frac{1}{3}$

Quiz 9 Thursday

① $f(x,y)=x^2y+xy^3$, $(2,1)$

$z_0=f(2,1)=2^2(1)+2(1)^3=4+2=6$

$z-z_0=f_x(x-2)+f_y(y-1)$

$\boxed{z-6=5(x-2)+10(y-1)}$

$f_x=2xy+y^3=4+1=5$

$f_y=x^2+3xy^2=4+6=10$

② $\frac{\partial F}{\partial y}=\cancel{\frac{\partial F}{\partial u}\frac{\partial u}{\partial y}}^{\,0}+\frac{\partial F}{\partial v}\frac{\partial v}{\partial y}$

$F(u,v)=e^{u+v}$ $\quad \frac{\partial F}{\partial u}=e^{u+v}$ $\quad \frac{\partial F}{\partial v}=e^{u+v}$

$u=x^2$, $\frac{\partial u}{\partial y}=0$ $\quad v=xy$ $\quad \frac{\partial v}{\partial y}=x$

$=e^{u+v}x$

$=e^{x^2+xy}x$

Quiz 9 Tuesday

① Same as Quiz 9 Thursday #1

② $f(x,y,z) = z\sqrt{x+y}$ at $(8,4,5)$

$= z(x+y)^{1/2}$

$f_x = \frac{1}{2}z(x+y)^{-1/2} = \frac{5}{2}(8+4)^{-1/2} = \frac{5}{2\sqrt{12}} = \frac{5}{4\sqrt{3}}$

$f_y = \frac{1}{2}z(x+y)^{-1/2} = \frac{5}{2}(8+4)^{-1/2} = \frac{5}{4\sqrt{3}}$

$f_z = (x+y)^{1/2} = (8+4)^{1/2} = \sqrt{12} = 2\sqrt{3}$

$L(x,y,z) = f_x(x-8) + f_y(y-4) + f_z(z-5)$

$= \boxed{\frac{5}{4\sqrt{3}}(x-8) + \frac{5}{4\sqrt{3}}(y-4) + 2\sqrt{3}(z-5)}$