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Math 31B Worksheet
Week 1

$(a^m)^n = a^{mn}$

1. Compute each of the following without a calculator:

   (a) $\dfrac{(3^7)^4}{(3^4)^7}$ *(Hint: Simplify the top and bottom.)* $\dfrac{3^{28}}{3^{28}} = 1$

$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

   (b) $8^{5/3}$ *(Hint: Use the fact that $\frac{5}{3} = 5 \cdot \frac{1}{3}$ to rewrite this in* two *different ways. One of them is easy to compute.)*

   $8^{5/3} = (\sqrt[3]{8})^5 = 2^5 = 32$

2. The conclusion from the end of Monday's lecture was

$$\text{if } f(x) = e^x, \text{ then } f'(x) = \underline{\ e^x\ }.$$

Chain rule $[f(g(x))]' = f'(g(x))\,g'(x)$

* Note: using chain rule, $(e^{g(x)})' = e^{g(x)} \cdot g'(x)$

3. For this problem, let $f(x) = x + e^{-x}$.

   (a) Find $f'(x)$. $f'(x) = 1 + \underbrace{e^{-x} \cdot (-1)}_{\text{chain rule}}$

   $= \boxed{1 - e^{-x}}$

   (b) f has one critical point. Find it.

   Set $f'(x) = 0$: $1 - e^{-x} = 0$

   $e^{-x} = 1$

   $x = 0$

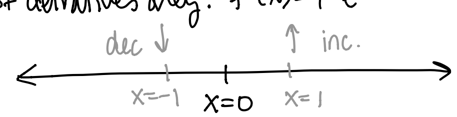
   (c) Review from 31A: Explain briefly the two ways to classify a critical point as a local maximum, local minimum, or neither.

   One way: 1st derivatives only. Determine where $f'(x) > 0$ (i.e. increasing) and $f'(x) < 0$ (decreasing). If the function changes from inc. to dec. $\Rightarrow$ local max; dec. to inc. $\Rightarrow$ local min. No change $\Rightarrow$ neither.

   Another way: 2nd derivatives. If $f''(x) > 0 \Rightarrow$ concave up $\cup \Rightarrow$ local min
   " $f''(x) < 0 \Rightarrow$ concave down $\cap \Rightarrow$ local max
   " $f''(x) = 0 \Rightarrow$ neither, inflection point

   (d) Use either of the two methods to classify the critical point of f.

1st derivatives only: $f'(x) = 1 - e^{-x}$



Test points.
$x = -1$: $f'(-1) = 1 - e^{-(-1)} = 1 - e < 0$
$x = 1$: $f'(1) = 1 - e^{-1} = 1 - \frac{1}{e} > 0$
dec to inc means $\boxed{\text{local min at } x = 0}$

2nd derivative: $f'(x) = 1 - e^{-x}$
$f''(x) = 0 - \underbrace{e^{-x}(-1)}_{\text{chain rule}} = e^{-x}$

Plug in the critical pt $x = 0$: $f''(0) = e^{-0} = 1 > 0$
concave up $\cup$
$\boxed{\text{local min at } x = 0}$

4. Rewrite as the logarithm of a single expression: $2\ln(6) - 3\ln(2)$

$$2\ln(6) - 3\ln 2 = \ln(6^2) - \ln(2^3) = \ln\left(\frac{6^2}{2^3}\right) = \ln\left(\frac{36}{8}\right) = \boxed{\ln\left(\frac{9}{2}\right)}$$

5. Consider the equation $\ln(x^2+1) - 3\ln(x) = \ln(2)$

(a) Simplify the left-hand side into a logarithm of a single expression.

$$\ln(x^2+1) - \ln(x^3) = \ln 2$$

$$\ln\left(\frac{x^2+1}{x^3}\right) = \ln 2$$

(b) Then eliminate the logarithms by exponentiating both sides. You should be able to easily simplify the result into a polynomial.

$$e^{\ln\left(\frac{x^2+1}{x^3}\right)} = e^{\ln 2}$$

$$\frac{x^2+1}{x^3} = 2$$

$$x^2+1 = 2x^3$$

$$\boxed{2x^3 - x^2 - 1 = 0}$$

(c) The polynomial you got in the above step is a cubic (degree 3), so it's a bit tricky to solve. One of the roots is easy to guess. Once you know that one, how does that allow you to find the other roots?

Guess a value for x so that $2x^3 - x^2 - 1 = 0$.

If we guess $x=1$: $2(1)^3 - 1^2 - 1 = 2 - 1 - 1 = 0$ ✓ This means $x-1$ is a factor of $2x^3 - x^2 - 1$.

What are the other factors? Use long division or synthetic division.

Long division:

$$\begin{array}{r l}
 & 2x^2 + x + 1 \\
x-1 & \overline{)\,2x^3 - x^2 + 0x - 1} \\
 & -(2x^3 - 2x^2) \\
\hline
 & x^2 + 0x - 1 \\
 & -(x^2 - x) \\
\hline
 & x - 1 \\
 & -(x-1) \\
\hline
 & 0
\end{array}$$

Synthetic division:

$$\begin{array}{r|rrrr}
1 & 2 & -1 & 0 & -1 \\
 & & 2 & 1 & 1 \\
\hline
 & 2 & 1 & 1 & \boxed{0}
\end{array}$$

$\Rightarrow 2x^2 + x + 1$

Therefore $2x^3 - x^2 - 1 = 0 \Rightarrow (x-1)(2x^2+x+1) = 0 \Rightarrow x - 1 = 0$ or $2x^2 + x + 1 = 0$

$x = 1$

quadratic formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(2)}$$

$$\boxed{x = \frac{-1 \pm i\sqrt{7}}{4}}$$