WEEK 2 WORKSHEET SOLUTIONS

Written by Victoria Kala
vtkala@math.ucla.edu

① (a) $f(x) = x^{\cos(x)}$ Take ln of both sides: $\ln f(x) = \ln\left(x^{\cos x}\right)$

$\ln f(x) = \cos x \ln x$

Take derivative of both sides: $(\ln f(x))' = (\cos x \ln x)'$

by chain rule $\longrightarrow \frac{1}{f(x)} \cdot f'(x) = -\sin x \ln x + \cos x \cdot \frac{1}{x}$

$\Rightarrow f'(x) = f(x)\left[-\sin x \ln x + \frac{\cos x}{x}\right]$

$\Rightarrow \boxed{f'(x) = x^{\cos x}\left[-\sin x \ln x + \frac{\cos x}{x}\right]}$

(b) $f(x) = x^{\cos x} = e^{\ln(x^{\cos x})} = e^{\cos x \ln x}$ ← rewriting the function

Now take derivative using chain rule: $f'(x) = e^{\cos x \ln x}(\cos x \ln x)'$

$= e^{\cos x \ln x}\left(-\sin x \ln x + \cos x \cdot \frac{1}{x}\right)$

$= \boxed{x^{\cos x}\left(-\sin x \ln x + \frac{\cos x}{x}\right)}$ ← same as (a)!

(c) $(b^x)' = \ln b \cdot b^x$ only applies if b is a constant. Here $f(x)$ is a function of x raised to a function of x.

② (a) $\lim_{x\to 8} \frac{x^{4/3} - x - 8}{x^{1/3} - 2} = \frac{8^{4/3} - 8 - 8}{8^{1/3} - 2} = \frac{2^4 - 16}{2-2} = \frac{0}{0}$ L'Hôpital's rule applies!

$\overset{LH}{=} \lim_{x\to 8} \frac{\frac{4}{3}x^{1/3} - 1}{\frac{1}{3}x^{-2/3}} = \frac{\frac{4}{3}(8)^{1/3} - 1}{\frac{1}{3}(8)^{-2/3}} = \frac{\frac{4}{3}\cdot 2 - 1}{\frac{1}{3}\cdot 2^{-2}} = \frac{\frac{5}{3}}{\frac{1}{12}} = \frac{5}{3}\cdot\frac{12}{1} = 20$

(b) $\lim_{x\to 1} \frac{x^2}{\ln x} = \frac{1}{0}$ L'Hôpital's rule does NOT apply

$= \infty$ because a finite # divided by a small # will "blow up"

(c) $\lim_{x\to\infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$ L'Hôpital's rule applies!

$\overset{LH}{=} \lim_{x\to\infty} \frac{2x}{e^x} = \frac{\infty}{\infty}$ Use L'Hopital's rule again

$\overset{LH}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0$

③ Short answer: no.

$\lim_{x\to 0^+} \frac{x}{\ln x} = \frac{0}{-\infty}$ L'Hôpital's rule does NOT apply

$= 0$ because a small # divided by a large negative number will be small

$\lim_{x\to 0^+} \frac{\ln x}{\sin x} = \frac{-\infty}{0}$ L'Hôpital's rule does NOT apply

$= -\infty$ because a large negative number divided by a small number will "blow up"

④ (a) 0^0

(b) $y = \lim_{x\to 0^+} x^{\sin x}$. Take ln of both sides: $\ln y = \lim_{x\to 0^+} \ln\left(x^{\sin x}\right) = \lim_{x\to 0^+} \sin x \ln x$

(c) Need to write as a fraction.

$\ln y = \lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\underbrace{\frac{1}{\sin x}}_{(\sin x)^{-1}\text{, can also use } \csc x}} \overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-(\sin x)^{-2}\cos x} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}} = \lim_{x\to 0^+} \frac{-\sin^2 x}{x\cos x} = \frac{0}{0}$

Use L'Hopital's rule again: $\ln y = \lim_{x\to 0^+} \frac{-\sin^2 x}{x\cos x} \overset{LH}{=} \lim_{x\to 0^+} \frac{-2\sin x\cos x}{\cos x - x\sin x} = \frac{0}{1} = 0$

$\Rightarrow \ln y = 0 \Rightarrow y = e^0 = 1$

(d) $0^1 = 0$ But $1^0 = 1$ $a^0 = 1$ for $a \neq 0$, but $0^a = 0$ for all finite a.

$0^{1/2} = 0$ $0.1^0 = 1$ We don't know if 0^0 is 1 or 0 $\Rightarrow$ indeterminate

$0^{1/100} = 0$ $0.00001^0 = 1$

etc. etc.