Math 32B Integration Review "Quiz"

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Evaluate the following integrals:

Substitution

1. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{1}{u} du = -\ln|u| + C = \boxed{-\ln|\cos x| + C}$

$u = \cos x$
$du = -\sin x \, dx$

2. $\int \frac{1}{x} dx = \boxed{\ln|x| + C}$

partial fraction decomposition

3. $\int \frac{1}{4 - x^2} dx$ $\quad \frac{1}{4-x^2} = \frac{1}{(2-x)(2+x)} = \frac{A}{2-x} + \frac{B}{2+x} \Rightarrow 1 = A(2+x) + B(2-x)$

If $x = -2$ $\quad 1 = A \cdot 0 + B \cdot 0 \longrightarrow B = 1/4$

If $x = 2$ $\quad 1 = A \cdot 4 + B \cdot 0 \longrightarrow A = 1/4$

Then $\int \frac{1}{4-x^2} dx = \int \left(\frac{1/4}{2-x} + \frac{1/4}{2+x} \right) dx = \boxed{-\frac{1}{4}\ln|2-x| + \frac{1}{4}\ln|2+x| + C}$

Chain rule!

4. $\int \frac{1}{x^2 + 1} dx = \boxed{\arctan x + C}$

Integration by parts
$\int u\,dv = uv - \int v\,du$

5. $\int x\cos x\,dx$ = $x\sin x - \int \sin x\,dx$ = $\boxed{x\sin x + \cos x + C}$

$u = x$ $\quad dv = \cos x\,dx$

$du = dx$ $\quad v = \sin x$

6. $\int \left(\sin x - e^{3x} + \frac{1}{x^3} + 5x^2\right) dx$ = $\int (\sin x - e^{3x} + x^{-3} + 5x^2)\,dx = -\cos x - \frac{1}{3}e^{3x} - \frac{x^{-2}}{2} + \frac{5}{3}x^3 + C$

$= \boxed{-\cos x - \frac{1}{3}e^{3x} - \frac{1}{2x^2} + \frac{5}{3}x^3 + C}$

7. $\int \sin^2 x\,dx$ = $\frac{1}{2}\int (1 - \cos 2x)\,dx = \boxed{\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C}$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

8. $\int \cos^2 x\,dx$ = $\frac{1}{2}\int (1 + \cos 2x)\,dx = \boxed{\frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C}$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$