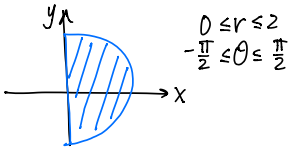
WEEK 3 Polar Coordinates Example

Ex Evaluate $\iint_D e^{-x^2-y^2}\,dA$ where D is the region bounded by the semicircle $x=\sqrt{4-y^2}$ and the y-axis.



$0 \le r \le 2$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$\int_{-\pi/2}^{\pi/2}\int_0^2 e^{-r^2}\, r\,dr\,d\theta \qquad u=-r^2 \qquad du=-2r\,dr$$

$$\int_{-\pi/2}^{\pi/2}\left(-\frac{1}{2}\int_0^{-4} e^u\,du\right)d\theta = \int_{-\pi/2}^{\pi/2} -\frac{1}{2}e^u\Big|_{u=0}^{-4}\,d\theta = \int_{-\pi/2}^{\pi/2} -\frac{1}{2}\left(e^{-4}-1\right)d\theta$$

$$= -\frac{1}{2}\left(e^{-4}-1\right)\theta\Big|_{\theta=-\pi/2}^{\pi/2} = \boxed{-\frac{1}{2}\left(e^{-4}-1\right)\pi}$$