

"What series test should I use?"

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* Deciding what test to use is usually trial and error, but below are my tips I have collected over the years

* If you try a bunch of tests and it is just not working, double check w/ Divergence Test

* Ratio Test is my "go to"

<u>Terms in series</u>	<u>Examples</u>	<u>Test to use</u>
$(\text{stuff})^n$	$\sum_{n=1}^{\infty} e^n, \sum_{n=1}^{\infty} \frac{2^n}{7^n}, \sum_{n=1}^{\infty} \left(\frac{2n+1}{n}\right)^n$	Divergence Test Geometric Series Root Test
$\ln(\text{stuff})$	$\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}, \sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}, \sum_{n=1}^{\infty} \frac{1}{n! \ln n}$	Divergence Test Integral Test Limit Comparison ($\ln n < n^a$ for all $a > 0$) Ratio Test
powers of n	$\sum_{n=2}^{\infty} \frac{n^3 + n^{1000}}{\sqrt{n^5 - 7n}}, \sum_{n=1}^{\infty} \frac{n^{1/2} - n^{3/2}}{n+1}$	Divergence Test Limit Comparison
$(-1)^n$	$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}, \sum_{n=1}^{\infty} \frac{(-1)^n n}{3^n}$	Divergence Test Alternating Series Test Ratio Test Root Test Geometric Series
$\cos, \sin$	$\sum_{n=1}^{\infty} \frac{\cos n}{n^2}, \sum_{n=1}^{\infty} \frac{\sin(n^{1000}-1)}{n^{1111}}$	Divergence Test Direct Comparison
Factorials, $n!$	$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}, \sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}, \sum_{n=1}^{\infty} \frac{e^n}{n!}$	Divergence Test Ratio Test
A "hodge podge" of terms	$\sum_{n=1}^{\infty} \frac{n^n}{n!(n-1)}, \sum_{n=1}^{\infty} \frac{(-5)^n \ln n}{(2n+1)!}$	Divergence Test Ratio Test
"Weird looking" series, series that you haven't seen before	$\sum_{n=1}^{\infty} e^{\sqrt{n}}, \sum_{n=1}^{\infty} \tan(n), \sum_{n=1}^{\infty} \frac{3^n}{3^n + 2^n}$	Divergence Test