

## Math 32B Practice Problems II

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1. Evaluate  $\iint_R xydA$  where  $R$  is the region in the first quadrant bounded by the lines  $y = x$ ,  $y = 3x$  and the hyperbolas  $xy = 1$ ,  $xy = 3$ . Use the transformation  $y = u/v$ ,  $y = v$ .
2. Determine whether or not  $\mathbf{F}$  is a conservative vector field. If it is, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .
  - (a)  $\mathbf{F}(x, y) = e^x \cos y \mathbf{i} + e^x \sin y \mathbf{j}$
  - (b)  $\mathbf{F}(x, y) = (\ln y + 2xy^3) \mathbf{i} + \left(3x^2y^2 + \frac{x}{y}\right) \mathbf{j}$
3. Evaluate the line integral  $\int_C xyzds$  where  $C$  is the curve parametrized by  $x = 2 \sin t$ ,  $y = t$ ,  $z = -2 \cos t$ ,  $0 \leq t \leq \pi$ .
4. Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $C$  is given by the vector function  $\mathbf{r}(t) = t\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}$ ,  $0 \leq t \leq \pi$  and  $\mathbf{F} = z\mathbf{i} + y\mathbf{j} - x\mathbf{k}$ .
5. Let  $\mathbf{F}(x, y, z) = y^2 \cos z \mathbf{i} + 2xy \cos z \mathbf{j} - xy^2 \sin z \mathbf{k}$ ,  $C$  be the curve parametrized by  $\mathbf{r}(t) = t^2\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ ,  $0 \leq t \leq \pi$ .
  - (a) Show that  $\mathbf{F}$  is conservative.
  - (b) Find a function  $f$  such that  $\mathbf{F} = \nabla f$ .
  - (c) Use (b) to calculate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  along the given curve  $C$ .
6. Evaluate  $\iint_S \mathbf{F} \cdot d\mathbf{S}$  where  $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$  and  $S$  is the boundary of the solid region  $E$  enclosed by the paraboloid  $z = 1 - x^2 - y^2$  and the plane  $z = 0$ .