Math 32B Week 6 Worksheet

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1. Evaluate the integral $\iint_R e^{(x+y)/(x-y)} dA$ where R is the trapezoidal region with vertices $(1,0), (2,0), (0,-2), (0,-1)$. Use the transformation $x = \frac{1}{2}(u+v), y = \frac{1}{2}(u-v)$. *Hint:* The region of integration S in the uv-plane is $\{(u,v) : 1 \leq v \leq 2, -v \leq u \leq v\}$.

2. Let $\mathbf{F} = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$.

   (a) Calculate $\text{div}(\mathbf{F})$.

   (b) Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a potential function f such that $\mathbf{F} = \nabla f$.

3. Write the parametrization for the following problems. *Hint*: use polar coordinates for circle and disk parametrizations.

   (a) The line from $(1, 1)$ to $(4, 2)$

   (b) The line from $(4, 2)$ to $(1, 1)$

   (c) The line from $(2, -4, 5)$ to $(7, 9, -1)$

   (d) The arc of the parabola $x = 4 - y^2$ from $(-5, -3)$ to $(0, 2)$

   (e) The upper half of the unit circle $x^2 + y^2 = 1$ starting at $(1, 0)$ and ending at $(-1, 0)$

   (f) The upper half of the unit circle $x^2 + y^2 = 1$ starting at $(-1, 0)$ and ending at $(1, 0)$

   (g) The unit circle centered at the origin, oriented clockwise

   (h) The circle of radius 3 centered at $(-1, 2)$

   (i) The disk $x^2 + y^2 = 4$ on the plane $z = -5$, oriented counterclockwise

   (j) The disk $x^2 + z^2 = 9$ on the plane $y = -1$