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Math 31B Worksheet
Week 8

1. Finish the formula for integration by parts:

$$\int u\,dv = uv - \int v\,du$$

2. Write down what the letters in the LIATE acronym stand for:

L: Logarithms

I: Inverse Trig

A: Algebraic (e.g. $x, x^2, \ldots$)

T: Trig

E: Exponential

3. Compute $\int e^{-x}\sin(2x)\,dx$

(Hint: Do integration by parts twice, simplify, then note that the last integral you're left with is the same as the original problem!)

$u = \sin(2x)$ $dv = e^{-x}dx$
$du = 2\cos(2x)dx$ $v = -e^{-x}$

$$\int e^{-x}\sin(2x)\,dx = -e^{-x}\sin(2x) + \underbrace{\int 2\cos(2x)e^{-x}\,dx}_{\text{parts again}}$$

$u = 2\cos(2x)$ $dv = e^{-x}dx$
$du = -4\sin(2x)dx$ $v = -e^{-x}$

$$= -e^{-x}\sin(2x) - 2e^{-x}\cos(2x) - \underbrace{\int 4e^{-x}\sin(2x)\,dx}_{\text{what we started with}}$$

$$5\int e^{-x}\sin(2x)\,dx = -e^{-x}\sin(2x) - 2e^{-x}\cos(2x) \longrightarrow \int e^{-x}\sin(2x)\,dx = \boxed{\frac{1}{5}\left(-e^{-x}\sin(2x) - 2e^{-x}\cos(2x)\right) + C}$$

4. Compute $\int \sin(\sqrt{x})\,dx$

(Hint: First, do a u-substitution (the obvious one). Then integrate by parts.)

$w = \sqrt{x}$ $dw = \frac{1}{2\sqrt{x}}dx$
$\Rightarrow 2\sqrt{x}\,dw = dx$
$\Rightarrow 2w\,dw = dx$

$$\int \sin(\sqrt{x})\,dx = \int \sin(w)\cdot 2w\,dw$$

parts: $u = 2w$ $dv = \sin w\,dw$
$du = 2dw$ $v = -\cos w$

$$= -2w\cos w + \int 2\cos w\,dw$$
$$= -2w\cos w + 2\sin w + C$$
$$= \boxed{-2\sqrt{x}\cos(\sqrt{x}) + 2\sin(\sqrt{x}) + C}$$

5. Consider the integral $\int \frac{x\,e^x}{(1+x)^2}\,dx$

(a) Following the LIATE acronym, what would you choose for u? Try doing integration by parts with this u, and see what happens.

$u = \frac{x}{(1+x)^2}$ $\quad dv = e^x\,dx$

$du =$ a mess $\quad v = e^x$

(b) Now try doing integration by parts using $u = x\,e^x$.

$u = xe^x$ $\quad dv = \frac{1}{(1+x)^2}dx$ power rule

product rule $du = (e^x + xe^x)dx$ $\quad v = \frac{-1}{1+x}dx$

$= e^x(1+x)dx$

$$\int \frac{xe^x}{(1+x)^2}dx = \frac{-1}{1+x}\cdot xe^x + \int \frac{+1}{1+x}\cdot e^x(1+x)dx = \frac{-xe^x}{1+x} + \int e^x dx = \boxed{\frac{-xe^x}{1+x} + e^x + C}$$

The point is that LIATE doesn't always work!

6. Compute $\int_0^1 \arctan(x)\,dx$
(Hint: According to LIATE, what should u be? Then dv is just. . . ? Also, since this is a definite integral, your final answer must be a number!)

$u = \arctan x$ $\quad dv = dx$

$du = \frac{1}{1+x^2}dx$ $\quad v = x$

$$\int_0^1 \arctan x\,dx = x\arctan x\Big|_0^1 - \int_0^1 \frac{x}{1+x^2}dx$$

$w = 1+x^2$

$dw = 2x\,dx$

$\frac{1}{2}dw = x\,dx$

when $x=0, u=1$

$x=1, u=2$

$$= x\arctan x\Big|_0^1 - \frac{1}{2}\int_1^2 \frac{1}{w}dw$$

$$= x\arctan x\Big|_0^1 - \frac{1}{2}\ln|w|\Big|_1^2$$

$$= 1\arctan 1 - \cancel{0\arctan 0}^{\,0} - \frac{1}{2}(\ln 2 - \cancel{\ln 1}^{\,0})$$

$$= \boxed{\frac{\pi}{4} - \frac{1}{2}\ln 2}$$