Math 32B Practice Problems III

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1. Evaluate $\int_C xydx + x^2dy$ where C is the rectangle with vertices $(0,0),(3,0),(3,1),(0,1)$ oriented counterclockwise.

2. Use Stokes' Theorem to evaluate $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$ where $\mathbf{F} = x^2z^2\mathbf{i} + y^2z^2\mathbf{j} + xyz\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies in the cylinder $x^2 + y^2 = 4$.

3. Consider the vector field $\mathbf{F}(x,y,z) = yz\mathbf{i} + 2xz\mathbf{j} + e^{xy}\mathbf{k}$ where C is the circle $x^2 + y^2 = 16, z = 5$ oriented counterclockwise when viewed from above.

   (a) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ by finding an appropriate parametrization vector $\mathbf{r}(t)$.

   (b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Stokes' Theorem, and verify it is equal to your solution in part (a).

4. Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x,y,z) = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$ where E is the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (*Note*: to verify the theorem is true you need to show calculate both $\iint_S \mathbf{F} \cdot d\mathbf{S}$ and $\iiint_E \text{div}(\mathbf{F})dV$ and show they are equal.)

5. Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = (\cos z + xy^2)\mathbf{i} + xe^{-z}\mathbf{j} + (\sin y + x^2z)\mathbf{k}$, and S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.