April 23, 2019

Q: If $A^2 = I$, is A always invertible? (From Chapter questions)

A: **Yes.**

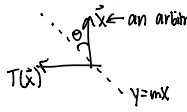
A (square matrix) is invertible $\iff$ there is a matrix B such that $AB = I$, $BA = I$.

We know $I = A^2 \Rightarrow I = AA \Rightarrow$ A is its own inverse! So A is invertible.

(this is your B!)

Q: Given $T(\vec{x}) = A\vec{x}$ a reflection about a line $y = mx$, how to find m? * a lot of ways to solve this

A: one way



$\vec{x}$ ← an arbitrary vector you can try

θ = angle between $\vec{x}$ and $y = mx$

2θ = " " " and $T(\vec{x})$ $\longrightarrow$ $\vec{x} \cdot T(\vec{x}) = \|\vec{x}\| \, \|T(\vec{x})\| \cos(2\theta)$

$\Rightarrow 2\theta = \cos^{-1}\left(\frac{\vec{x} \cdot T(\vec{x})}{\|\vec{x}\| \, \|T(\vec{x})\|}\right)$, divide by 2 to get θ

In class I tried to do projection of $\vec{x}$ onto line, easier to just rotate $\vec{x}$ by θ:

$\vec{u} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \vec{x}$ (multiply this out)

You will get a vector $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ ← run, ← rise $\rightarrow$ $m = \frac{u_2}{u_1}$

Another way: Similar to the above approach

$T(\vec{x}) = A\vec{x}$ Find $T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = A\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ multiply this out to get a vector