Math 33A − Week 2

Written by Victoria Kala

April 9, 2019

Name: _____

1. Let $A = \begin{pmatrix} 1 & 5 & 3 \\ 0 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -3 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix}$. Compute the following if defined. If not defined, state why.

   (a) $B + C$ (b) $A - 2B$

2. Give examples of a matrix A with rank 2 and appropriate sized vector b so that the system $Ax = b$ has

   (a) No solution   (b) One solution   (c) Infinitely many solutions

3. Let $x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $y = \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$. Compute $x \cdot y$.

4. Is $x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ a linear combination of $y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $z = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$?

5. Determine which of the following transformations are linear. If the transformation is linear, find the matrix A so that $T(x) = Ax$.

(a) $T(x_1) = x_1^2$

(b) $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \end{pmatrix}$

(c) $T(x_1, x_2, x_3) = (x_1 + 1, \; 5x_2 - x_3)$

(d) $T(x_1, x_2) = (x_1, \; 5x_2, \; x_1 - 3x_2, \; 3x_2 - x_1)$

6. (a) Let $x = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$. Find a, b, c so that

$$x = a\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(b) Suppose we know T is a linear transformation with $T\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}, T\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -7 \\ 4 \end{pmatrix}, T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$. What is $T(x)$?