Final Practice T/F Questions

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Name: ______________________________ Score: NA

Directions: You may use one 3" × 5" notecard, however no other outside resources such as books, notes, or calculators are allowed. Write your solutions in your bluebook and clearly mark the problems. If you solve a problem multiple times, cross out the work you do not want graded, otherwise you will receive little or no partial credit. Unless otherwise specified, numbers included in a solution are not to be approximated, but instead expressed as exact numbers (i.e., in terms of square roots, multiples of π, etc.).

Disclaimer: The content and level of difficulty of this quiz are not guaranteed to be in correlation with the midterm nor final examinations in any form.

Circle true (T) or false (F) for each statement below. If the statement is false, show why, provide a counterexample, or correct the statement (where possible).

1. T F If A and B are square matrices, then $\det(A+B) = \det A + \det B$.
2. T F If A and B are square matrices, then $\det(AB) = \det(A)\det(B)$.
3. T F If A and B are square matrices and AB is defined, then $\det(AB) = \det(A)\det(B)$.
4. T F All matrices have an inverse.
5. T F All square matrices have an inverse.
6. T F All square matrices with nonzero determinant have an inverse.
7. T F If A has eigenvalue λ, then A^k has eigenvalue $k\lambda$.
8. T F For any two matrices A, B, the product AB is always defined.
9. T F For any two matrices A, B the sum $A + B$ is always defined.
10. T F If A is an $m \times n$ matrix, the Rank-Nullity Theorem states $rank(A) + nullity(A) = m$.
11. T F The following system is consistent:

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 4x_1 - 8x_2 + 12x_3 &= 1 \end{aligned}$$

12. T F It is possible for a system of equations to have exactly two solutions.

13. T F The following system has a unique solution:

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned}$$

14. T F The vector $(3, -1)$ is a solution to the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 5 \\ -5 & 6 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 7 \\ 4 \\ -3 \end{pmatrix}$$

15. T F Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The formula for A^{-1} is

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} a & -b \\ -c & d \end{pmatrix}.$$

16. T F The general solution of $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 7 \\ -1 \\ -4 \end{pmatrix}$$

is $\mathbf{x} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{4}{3} \\ 0 \\ 1 \end{pmatrix} x_3$.

17. T F If A is diagonalizable then there exists a D and invertible P such that $A = PDP^{-1}$.

18. T F If A is similar to B, then A^k is similar to B^k.

19. T F The set of eigenvalues λ of A are the set of values such that $A - \lambda I$ is not invertible.

20. T F The following set of vectors are linearly independent:

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$$

21. T F The set of vectors in the previous problem (#17) span $\mathbb{R}^2$.

22. T F If A is an invertible matrix then $\det(A^{-1}) = \dfrac{1}{\det(A)}$.

23. T F All transformations are linear transformations.

24. T F The transformation defined by $T(\mathbf{x}) = 3\mathbf{x}$ is a linear transformation.

25. T F The matrix representation for the transformation $T(\mathbf{x}) = 3\mathbf{x}$ is $A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

26. T F If the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $null(A) = \{\mathbf{0}\}$.

27. T F If A and B are invertible matrices then $(AB)^{-1} = A^{-1}B^{-1}$.

28. T F If A and B are invertible matrices then $(AB)^{-1} = B^{-1}A^{-1}$.

29. T F If the product AB is defined, then $(AB)^T = A^T B^T$.

30. T F If the product AB is defined, then $(AB)^T = B^T A^T$.

31. T F The set $U = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 : x_1 \geq 0, x_2 \geq 0 \right\}$ is a subspace of $\mathbb{R}^2$.

32. T F The rank is the dimension of the null space.

33. T F The rank is the dimension of the column space.

34. T F The rank is the dimension of the row space.

35. T F The rank is the number of pivots of a reduced matrix.

36. T F The nullity is the number of free variables of a reduced matrix.

37. T F The determinant of a matrix is the product of its diagonal entries.

38. T F The determinant of a triangular (upper or lower) matrix is the product of its diagonal entries.

39. T F $\begin{vmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{vmatrix} = 20$

40. T F If A is a square matrix then $\det A = \det A^T$.

41. T F If $T(\mathbf{x}) = A\mathbf{x}$, then the kernel of T is the null space of A.

42. T F The pivot columns of a matrix form the column space of that matrix.

43. T F The following set of vectors forms a basis for $\mathbb{R}^3$:

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

44. T F The dot product of two vectors is always defined.

45. T F The matrix $\begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$ has the characteristic equation $(\lambda - 3)(\lambda + 7)$.

46. T F All square matrices are diagonalizable.

47. T F The norm of a vector $\mathbf{u}$ is given by $||\mathbf{u}|| = \mathbf{u} \cdot \mathbf{u}$.

48. T F The dot product is defined to be $\mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}|| ||\mathbf{v}|| \cos\theta$.

49. T F The matrix $U = \begin{pmatrix} 3/\sqrt{11} & -1/\sqrt{6} & -1/\sqrt{66} \\ 1/\sqrt{11} & 2/\sqrt{6} & -4/\sqrt{66} \\ 1/\sqrt{11} & 1/\sqrt{6} & 7/\sqrt{66} \end{pmatrix}$ is orthogonal.

50. T F The set of vectors below forms an orthogonal set:

$$\left\{ \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1/2 \\ -2 \\ 7/2 \end{pmatrix} \right\}$$