Math 4A: Quiz 1 Solution

October 9, 2015

Name: ____ Score: ________/10

Circle which section you attend: 8am 4pm 5pm 6pm 7pm

Directions: No outside resources such as books, notes, or calculators are allowed. You will have 15 minutes to complete this quiz.

Disclaimer: The content and level of difficulty of this quiz are not guaranteed to be in correlation with the midterm nor final examinations in any form.

For the problems below, consider the system

$$\begin{aligned} x_1 + x_3 &= 2 \\ x_2 + \alpha x_3 &= 0 \\ x_1 + 2x_2 + 13x_3 &= 0 \end{aligned}$$

where α is a constant.

1. (4 pt) For what values of α is the system
   (a) Inconsistent?
   (b) Consistent?

   Hint: use row operations to reduce the augmented matrix.

   Solution. We first write our system as an augmented matrix:

   .

   Our goal is to get something close to:

$$\left(\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & * & * \end{array}\right).$$

   There are many steps to do this. Here is one way:

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 1 & 2 & 13 & 0 \end{array}\right) \xrightarrow{-R_1+R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 2 & 12 & -2 \end{array}\right) \xrightarrow{-2R_2+R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 12-2\alpha & -2 \end{array}\right)$$

   You are welcome to stop here, or you can keep going. Let's stop here for now. To get an inconsistent system means we need to get a false statement. Notice if $12 - 2\alpha = 0$ then the last equation will be $0 = 2$, which is false. Thus $\alpha = 6$ will make our system inconsistent, and $\alpha \neq 6$ will make our system consistent. This is the solution.

Let's suppose we kept going with our matrix:

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 12-2\alpha & -2 \end{array}\right) \xrightarrow{\frac{1}{12-2\alpha}R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 1 & \frac{1}{\alpha-6} \end{array}\right)$$

We can only do this last step when $12-2\alpha \neq 0$ since we cannot divide by 0. This also gives us the same solution that $\alpha = 6$ gives us an inconsistent system and $\alpha \neq 6$ gives us a consistent system. □

2. (2 pt) Is it possible to find a value of α such that the system above has infinitely many solutions? Why or why not?

Proof. No. This system cannot have infinitely many solutions. We must have a consistent system in order to have infinitely many solutions, and so we know that $\alpha \neq 6$. Using this fact we can deduce the following:

- We can never get a row of 0's
- We will always have a pivot in the 3rd row, 3rd column
- We will never have a free variable

These facts all imply that we cannot have infinitely many solutions. If you mentioned any of these facts then you received full credit; you don't need all of them to come to the conclusion that there are not infinitely many solutions. □

3. (4 pt) For $\alpha = 7$, find the solution of the system using Gauss-Jordan (reduced row-echelon) elimination.

Proof. Our goal is to get to:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{array}\right).$$

We will continue using our work from the first part:

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 1 & \frac{1}{\alpha-6} \end{array}\right) \xrightarrow{\alpha=7} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{-7R_3+R2\to R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{-R_3+R1\to R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

Thus $x_1 = 1, x_2 = -7, x_3 = 1$. □