Math 4A: Quiz 2 Solutions

October 26, 2015

1. Circle the correct answer and show your work below: The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that maps $(1, 2)$ to $(-1, 1)$ and $(0, -1)$ to $(2, -1)$ will map $(1, 1)$ to...

   (A) $(1, 2)$ (B) $(1, 0)$ (C) $(2, -1)$ (D) $(2, 1)$ (E) $(1, 1)$

   Solution. The correct solution is **B**.

   Write the vector $(1, 1)$ as a linear combination of $(1, 2)$ and $(0, -1)$:

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

   You can solve for c_1, c_2 by observation or writing as an augmented matrix. It turns out that $c_1 = c_2 = 1$.

   Now use this information to find the transformation if $(1, 1)$:

$$\begin{aligned} T\begin{pmatrix} 1 \\ 1 \end{pmatrix} &= T\left[c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right] \\ &= c_1 T\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 T\begin{pmatrix} 0 \\ -1 \end{pmatrix} \\ &= 1 \cdot T\begin{pmatrix} 1 \\ 2 \end{pmatrix} + 1 \cdot T\begin{pmatrix} 0 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}$$

   Another way is to find a 2×2 matrix:

$$T(\mathbf{x}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mathbf{x}$$

   You can find a, b, c, d by using the information above for $T(1, 2)$ and $T(0, -1)$. □

2. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 3 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 2 & 4 \end{pmatrix}.$$

For each of the following operations, briefly explain whether or not they are defined. If they are defined, do it.

(a) $A + B$

Solution. Undefined. A is a 2×3 matrix and B is a 2×2 matrix. The size of A is not the same size of B so we cannot add the two matrices together. □

(b) AB

Solution. Undefined. A is a 2×3 matrix and B is a 2×2 matrix. The number of columns of A is not the same as the number of rows of B so we cannot multiply the two matrices in this order. □

(c) CB

Solution. Defined. C is a 3×2 matrix and B is a 2×2 matrix. The number of columns of C is the same as the number of rows of B. The resulting matrix should have size 3×2.

$$CB = \begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -10 & 2 \\ 10 & -8 \end{pmatrix}$$

□

(d) AC

Solution. Defined. A is a 2×3 matrix and C is a 3×2 matrix. The number of columns of A is the same as the number of rows of C. The resulting matrix should have size 2×2.

$$AC = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 20 & 11 \\ 47 & 23 \end{pmatrix}$$

□

(e) CA

Solution. Defined. C is a 3×2 matrix and A is a 2×3 matrix. The number of columns of C is the same as the number of rows of A. The resulting matrix should have size 3×3.

$$AC = \begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 3 & 9 & 15 \\ 18 & 24 & 30 \end{pmatrix}$$

□