Math 4A: Quiz 3 Solutions

October 30, 2015

1. Circle the correct answer and show your work below: If the matrices

$$\begin{pmatrix} 3 & -2 & -2 \\ -1 & 1 & 1 \\ 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{pmatrix}$$

are inverses of each other, what is the value of c?

(a) -3 (b) -2 (c) 0 (d) 2 (e) 3

Solution. The correct answer is **E**.

There are multiple ways to approach this problem. One way is to use row reduction to find the inverse of the first given matrix. Another way is to multiply the first matrix and the second matrix and obtain a system of equations with a, b, c. These will give you the correct solution, however, these methods take time. The fastest way is to multiply the second matrix and the first matrix:

$$\begin{pmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{pmatrix} \begin{pmatrix} 3 & -2 & -2 \\ -1 & 1 & 1 \\ 3 & -1 & -2 \end{pmatrix} = \begin{pmatrix} * & * & * \\ * & * & * \\ 6-c-3 & -4+c+1 & -4+c+2 \end{pmatrix}$$

Since these matrices are inverse of each other then their product should yield the identity matrix:

$$\begin{pmatrix} * & * & * \\ * & * & * \\ 3-c & c-3 & c-2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Setting the components equal to each other we have that $3 - c = 0, c - 3 = 0, c - 2 = 1$. Therefore we must have that $c = 3$, which is solution **E**. □

2. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$.

   (a) Find A^{-1}.

   Solution. Set up as an augmented matrix and use row operations to get reduced row echelon form:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-2R1+R2\to R2} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-R1+R3\to R3} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right) \xrightarrow{2R2+R3\to R3} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right)$$

$$\xrightarrow{-R3\to R3}\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right) \xrightarrow{3R3+R2\to R2}\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

$$\xrightarrow{-3R3+R1\to R1}\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right) \xrightarrow{-2R2+R1\to R1}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

Now that we have the identity matrix on the left hand side, the right hand side is the inverse matrix. That is,

$$A^{-1} = \begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$$

□

(b) Use what you found in part (a) to solve the system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 2x_1 + 5x_2 + 3x_3 &= 3 \\ x_1 + 8x_3 &= 17 \end{aligned}$$

Proof. We can write this system as a matrix equation $A\mathbf{x} = \mathbf{b}$:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 17 \end{pmatrix}$$

Now that we know A^{-1} exists (we found it), we can multiply by A^{-1} on both sides to get the equation $\mathbf{x} = A^{-1}\mathbf{b}$:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 17 \end{pmatrix} = \begin{pmatrix} -200 + 48 + 153 \\ 65 - 15 - 51 \\ 25 - 6 - 17 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Therefore the solution to the system above is $x_1 = 1, x_2 = -1, x_3 = 2$. □