Math 4A: Quiz 4 Solutions

November 7, 2015

1. (a) Use cofactor expansion to find the determinant of A where

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 3 & 1 & 2 & 2 \\ 1 & 0 & -2 & 1 \\ 2 & 0 & 0 & 1 \end{pmatrix}.$$

Solution. There are multiple ways to approach this problem. The second column has the most zeros in it:

$$\begin{vmatrix} 1 & 0 & 0 & -1 \\ 3 & 1 & 2 & 2 \\ 1 & 0 & -2 & 1 \\ 2 & 0 & 0 & 1 \end{vmatrix} = -0\begin{vmatrix} 3 & 2 & 2 \\ 1 & -2 & 1 \\ 2 & 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \\ 2 & 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 0 & -1 \\ 3 & 2 & 2 \\ 2 & 0 & 1 \end{vmatrix} + -0\begin{vmatrix} 1 & 0 & -1 \\ 3 & 2 & 2 \\ 1 & -2 & 1 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

We need to repeat the process as we have a 3×3 matrix. The second column has the most zeros in it:

$$\begin{vmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \\ 2 & 0 & 1 \end{vmatrix} = -0\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + (-2)\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$
$$= -2(1 \cdot 1 - (-1) \cdot 2)$$
$$= -6.$$

Thus $\det A = -6$.

Note: You received partial credit if you used a method other than cofactor expansion. □

(b) Does A^{-1} exist? Explain.

Solution. Yes, $\det A \neq 0$. □

2. Given that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$, find $\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$ using row reduction.

Solution. You can do this in two ways:

- Take the simpler matrix and use row operations to produce the more complicated matrix.
- Take the more complicated matrix and use row operations to produce the simpler matrix.

I personally think it is easier to start with the more complicated matrix.

We begin by multiplying the first row by $-\frac{1}{3}$; that is, $-\frac{1}{3}R1 \to R1$:

$$\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix} = -3\begin{vmatrix} a & b & c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$$

You can also think of this as factoring out the common -3 term from the first row.

Next, we add 4 times the second row to the third row; that is, $4R2 + R3 \to R3$:

$$-3\begin{vmatrix} a & b & c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix} = -3\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

Recall that adding a multiple of a row to another does not change the determinant.

Therefore

$$\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix} = -3\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3(-6) = 18.$$

Note: Some students just wrote $-3(-6) = 18$ as their solution. While this IS correct, I need to see you justify your solution by showing your work. No work receives little or no partial credit. You can do this by showing how the determinant changes with each row operation. □