1. You can look up your TARDIS code on Gauchospace.

2. Write as an augmented matrix and reduce:

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 4 \\ 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 2 \end{array}\right) \quad \xrightarrow{R1 \leftrightarrow R2} \quad \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 4 \\ 2 & 4 & 1 & 2 \end{array}\right) \quad \xrightarrow{-2R1 + R3 \rightarrow R3} \quad \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & -4 \end{array}\right)$$

$$\xrightarrow{R2 - R3 \rightarrow R2} \quad \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -4 \end{array}\right) \quad \xrightarrow{R1 - 2R2 \rightarrow R1} \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & -13 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -4 \end{array}\right)$$

Thus $x = -13, y = 8, z = -4$.

3. (a) There are **infinitely many solutions**. The third column is missing a pivot, therefore there is a free variable. Since there is a free variable there are infinitely many solutions.

   (b) **No solution**. Subtracting twice the second row from the third row yields a false statement.

   (c) **One solution**. Every column has a pivot, so there is a unique solution.

4. For $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ to be in the span of the given two vectors, there needs to be a linear combination

   of those two vectors equal to $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$:

$$a \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

   Check to see if the corresponding augmented matrix has a solution:

$$\left(\begin{array}{cc|c} 1 & 3 & 0 \\ -1 & 1 & 1 \\ 0 & 4 & 1 \end{array}\right) \quad \xrightarrow{R1 + R2 \rightarrow R2} \quad \left(\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 4 & 1 \\ 0 & 4 & 1 \end{array}\right) \quad \xrightarrow{R3 - R2 \rightarrow R3} \quad \left(\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

   This system has a solution. So **yes**, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ is in the span of the given vectors.

5. For the span of the given vectors to be equal to $\mathbb{R}^3$, we need the corresponding matrix to have a pivot in every row:

$$\left(\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{array}\right) \quad \xrightarrow{R3 - R1 \rightarrow R3} \quad \left(\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right) \quad \xrightarrow{R3 - R2 \rightarrow R3} \quad \left(\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

The last row is missing a pivot. So **no**, the span of the given vectors does not equal $\mathbb{R}^3$.

6. We need to find when the corresponding matrix augmented with the zero vector has a non-trivial solution:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 2 & k & 0 \end{array}\right) \quad \xrightarrow{R3 - R1 \to R3} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & k-1 & 0 \end{array}\right) \quad \xrightarrow{R3 - R2 \to R3} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & k-3 & 0 \end{array}\right)$$

If $k \neq 3$, we get the trivial solution. For $k = 3$ we get infinitely many solutions, which means there is a nontrivial solution. Thus $k = 3$.

7. There are two ways, outlined below:

(i) Method 1, find a matrix so that $T(\mathbf{x}) = A\mathbf{x}$:

$$A = \left(\begin{array}{cc} T(1,0) & T(0,1) \end{array}\right) = \left(\begin{array}{cc} 1 & 3 \\ 2 & 4 \end{array}\right)$$

Therefore

$$T\left(\begin{array}{c} 2 \\ -1 \end{array}\right) = \left(\begin{array}{cc} 1 & 3 \\ 2 & 4 \end{array}\right)\left(\begin{array}{c} 2 \\ -1 \end{array}\right) = \left(\begin{array}{c} 2-3 \\ 4-4 \end{array}\right) = \left(\begin{array}{c} -1 \\ 0 \end{array}\right).$$

(ii) Method 2, use linear combinations: Find a and b so that

$$\left(\begin{array}{c} 2 \\ -1 \end{array}\right) = a\left(\begin{array}{c} 1 \\ 0 \end{array}\right) + b\left(\begin{array}{c} 0 \\ 1 \end{array}\right)$$

By observation, $a = 2$ and $b = -1$. Take the transformation of both sides:

$$T\left(\begin{array}{c} 2 \\ -1 \end{array}\right) = T\left(2\left(\begin{array}{c} 1 \\ 0 \end{array}\right) - \left(\begin{array}{c} 0 \\ 1 \end{array}\right)\right)$$
$$= 2T\left(\begin{array}{c} 1 \\ 0 \end{array}\right) - T\left(\begin{array}{c} 0 \\ 1 \end{array}\right)$$
$$= 2\left(\begin{array}{c} 1 \\ 2 \end{array}\right) - \left(\begin{array}{c} 3 \\ 4 \end{array}\right)$$
$$= \left(\begin{array}{c} -1 \\ 0 \end{array}\right)$$