

Quiz 2 Solutions

June 28, 2016

1. Find the general solution of the ODE

$$ty' - y = t \cdot \ln t, t > 0.$$

Solution. Write equation in standard form by dividing by t :

$$y' - \frac{1}{t}y = \ln t$$

Find the integrating factor:

$$\mu = \exp \left(\int P(t)dt \right) = \exp \left(\int -\frac{1}{t}dt \right) = \exp(-\ln|t|) = e^{\ln|t|^{-1}} = |t|^{-1} = \frac{1}{|t|}$$

We can drop the absolute values since $t > 0$:

$$\mu = \frac{1}{t}$$

Multiply μ by our standard form equation:

$$\begin{aligned} \frac{1}{t}y' - \frac{1}{t^2}y &= \frac{\ln t}{t} \\ \frac{1}{t}y' + \left(\frac{1}{t}\right)'y &= \frac{\ln t}{t} \\ \left(\frac{1}{t}y\right)' &= \frac{\ln t}{t} \end{aligned}$$

Integrate:

$$\frac{1}{t}y = \int \frac{\ln t}{t} dt$$

Use the substitution $u = \ln t, du = \frac{1}{t}dt$:

$$\frac{1}{t}y = \frac{(\ln t)^2}{2} + C$$

Solve for y :

$$y = \frac{t(\ln t)^2}{2} + Ct$$

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