

## Quiz 4 Solutions

July 10, 2016

1. Consider the initial value problem:

$$(4 - t^2)y' + 2ty = \ln(t + 5), y(-3) = 1.$$

Determine (without solving the problem) the largest interval in which the solution of the IVP is certain to exist.

*Solution.* Write the equation in standard form:

$$y' + \frac{2t}{4 - t^2}y = \frac{\ln(t + 5)}{4 - t^2}.$$

The domain of  $P(t) = \frac{2t}{4 - t^2}$  is all  $t$  except  $t \neq \pm 2$ . Since  $\ln(t)$  has the domain  $t > 0$ , the domain of  $f(t) = \frac{\ln(t + 5)}{4 - t^2}$  is all  $t > -5$  and  $t \neq \pm 2$ . The domain of our equation can therefore be written as  $(-5, -2) \cup (-2, 2) \cup (2, \infty)$  in interval notation.

The point  $y(-3) = 1$  occurs when  $x = -3$  which is in the interval  $(-5, -2)$ . Therefore the Existence and Uniqueness theorem guarantees a unique solution in the interval  $(-5, -2)$ .  $\square$