Quiz 8 Solutions

July 24, 2016

1. Suppose a spring is horizontal and has one end attached to a wall and the other end attached to a 2kg mass. Suppose the friction constant is 6 N·s/m, and it requires 10N to stretch the spring 2m beyond its equilibrium position.

   (a) Set up a differential equation that describes this system. Let $x(t)$ denote the displacement from its equilibrium position at time t.

   Solution. The equation is of the form $mx''(t) + \gamma x'(t) + kx = 0$ where k is the spring constant found from Hooke's Law: $F = k\delta x$. We are given $m = 2$, $\gamma = 6$, and $k = \frac{10}{2} = 5$. Plug into the equation to get:

   $$2x'' + 6x + 5 = 0.$$

   □

   (b) Find the general solution of your differential equation.

   Solution. The equation in part (a) has the characteristic equation $2r^2 + 6r + 5 = 0$. Use the quadratic formula to get

   $$r = \frac{-6 \pm \sqrt{6^2 - 4(5)(2)}}{2(2)} = -\frac{3}{2} \pm \frac{i}{2}$$

   The general solution is therefore

   $$x = e^{-3t/2}\left(c_1 \cos\frac{t}{2} + c_2 \sin\frac{t}{2}\right).$$

   □

   (c) Is this system underdamped, overdamped, or critically damped?

   Solution. Underdamped since the roots to our characteristic equation were complex. □