Quiz 3 Solutions

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1. Consider the initial value problem:

$$(4-t^2)y' + 2ty = \ln(t+5), \quad y(-3) = 1.$$

Determine (without solving the problem) the largest interval in which the solution of the IVP is certain to exist.

Solution. Write the equation in standard form:

$$y' + \frac{2t}{4-t^2}y = \frac{\ln(t+5)}{4-t^2}$$

The domain of $P(t) = \frac{2t}{4-t^2}$ is $t \neq \pm 2$. Since $\ln(t)$ has the domain $t > 0$, the domain of $f(t) = \frac{\ln(t+5)}{4-t^2}$ is all $t > -5$ and $t \neq \pm 2$. The domain of our equation can therefore be written as $(-5,-2) \cup (-2,2) \cup (2,\infty)$ in interval notation.

The point $y(-3) = 1$ occurs when $x = -3$ and is in the interval $(-5,-2)$. Therefore the Existence and Uniqueness theorem guarantees a unique solution in the interval $(-5,-2)$. $\square$