Quiz 4 Solutions

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1. Consider the second order differential equation:

$$y'' + 4y' + 3y = 0.$$

   (a) find the general solution $c_1y_1(t) + c_2y_2(t)$ of the equation, and compute the Wronskian to show that y_1, y_2 form a fundamental set of solutions.

   (b) What is the solution if we impose the initial condition $y(0) = 3, y'(0) = -7$?

Solution. (a) The characteristic equation is

$$m^2 + 4m + 3 = 0 \quad \Rightarrow \quad (m+1)(m+3) = 0 \quad \Rightarrow \quad m = -1, -3$$

The solution is therefore $y = c_1e^{-t} + c_2e^{-3t}$.

We can check our solution by plugging it into the original equation. First find the derivatives:

$$\begin{aligned} y &= c_1e^{-t} + c_2e^{-3t} \\ y' &= -c_1e^{-t} - 3c_2e^{-3t} \\ y'' &= c_1e^{-t} + 9c_2e^{-3t} \end{aligned}$$

Plug in:

$$\begin{aligned} y'' + 4y' + 3y &= c_1e^{-t} + 9c_2e^{-3t} + 4(-c_1e^{-t} - 3c_2e^{-3t}) + 3(c_1e^{-t} + c_2e^{-3t}) \\ &= 0 \end{aligned}$$

So our solution is indeed a solution.

We now calculate the Wronskian. Set $y_1 = e^{-t}, y_2 = e^{-3t}$ (you can also switch these, your Wronskian will just differ by a sign). Then the Wronskian is given by

$$W(e^{-t}, e^{-3t}) = \begin{vmatrix} e^{-t} & e^{-3t} \\ -e^{-t} & -3e^{-3t} \end{vmatrix} = -3e^{-4t} + e^{-4t} = -2e^{-4t} \neq 0$$

Therefore our solution is $y = c_1e^{-t} + c_2e^{-3t}$ is a fundamental solution.

(b) When $t = 0$ we have that $y = 3$ and $y' = 7$. We then get the system of equations

$$\begin{cases} 3 &= c_1 + c_2 \\ -7 &= -c_1 - 3c_2 \end{cases}$$

which has the solution $c_1 = 1, c_2 = 2$. Our solution to the differential equation with the given initial condition is then $y = e^{-t} + 2e^{-3t}$.

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