Math 4B: Review "Quiz" Solutions

Written by Victoria Kala

June 28, 2017

1. (a) Use the power rule: $\frac{d}{dx}x^n = nx^{n-1}$ for $n \neq 0$. The solution is
$$\frac{d}{dx}x^{101} = 101x^{100}.$$
(b) Use the product rule: $(fg)' = f'g + fg'$. We then have that
$$\frac{d}{dx}(xe^{2x}) = x'e^{2x} + x(e^{2x})' = e^{2x} + 2xe^{2x}.$$
(c) Use the chain rule: $[f(g(x))]' = f'(g(x))g'(x)$. We then have that
$$(\arcsin(3x))' = \frac{1}{\sqrt{1-(3x)^2}} \cdot (3x)' = \frac{3}{\sqrt{1-9x^2}}.$$

2. (a) Use a u-substitution. Let $u = \cos x$. Then $du = -\sin x dx$, and
$$\int \tan x dx = \int \frac{\sin x}{cos x} dx = \int -\frac{1}{u} du = -\ln|u| + C = -\ln|\cos x| + C.$$
This solution can also be written as $\ln|\sec x| + C$.
(b) $\int \frac{1}{x} dx = \ln|x| + C$.
(c) We will use partial fraction decomposition to evaluate the integral. The denominator can be factored as $(2-x)(2+x)$. We will then find an A, B such that
$$\frac{1}{(2-x)(2+x)} = \frac{A}{2-x} + \frac{B}{2+x}.$$
Multiply both sides by $(2-x)(2+x)$ to get
$$\begin{aligned} 1 &= A(2+x) + B(2-x) \\ 0x + 1 &= 2A + Ax + 2B - Bx \\ 0x + 1 &= (A-B)x + 2A + 2B \end{aligned}$$
Compare the x terms: there are 0 x terms on the left hand side, there are $A - B$ x terms on the right hand side, hence
$$0 = A - B$$
Compare the constant terms: there is a 1 on the left hand side, and there is a $2A + 2B$ term on the right hand side, hence
$$1 = 2A + 2B.$$
Solving this system of equations yields $A = \frac{1}{4}, B = \frac{1}{4}$. Thus
$$\frac{1}{4-x^2} = \frac{1/4}{2-x} + \frac{1/4}{2+x},$$
and
$$\int \frac{1}{4-x^2} dx = \int \left(\frac{1/4}{2-x} + \frac{1/4}{2+x} \right) dx = -\frac{1}{4}\ln|2-x| + \frac{1}{4}\ln|2+x| + C.$$

(d) $\int \frac{1}{x^2+1} dx = \arctan x + C$.

(e) Use integration by parts: $\int u dv = uv - \int v du$. Take $u = x, dv = \cos x dx$. Then $du = 1dx$ and $v = \sin x$, and

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C.$$

(f) Rewrite $\frac{1}{x^3}$ as x^{-3} to use power rule and then simplify:

$$\int \left(\sin x - e^{3x} + x^{-3} + 5x^2\right) dx = -\cos x - \frac{1}{3}e^{3x} - \frac{1}{2x^2} + \frac{5}{3}x^3 + C.$$