Math 6A: Line Integral "Quiz"

April 28, 2016

Name: _____ Score: NA

Directions: Open book, open note, open neighbor.

Disclaimer: The content and level of difficulty of this quiz are not guaranteed to be in correlation with the midterm nor final examinations in any form.

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1. Evaluate $\int_C xydx+x^2dy$ where C is the rectangle with vertices $(0,0),(3,0),(3,1),(0,1)$ oriented counter clockwise.

2. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = \sin x\mathbf{i} + \cos y\mathbf{j} + xz\mathbf{k}$ and C is given by the parametrization $\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}, 0 \leq t \leq 1$.

3. A thin wire in the shape of a curve C with linear density $\rho(x, y)$ has **mass**

$$m = \int_C \rho(x, y) ds$$

and **center of mass** $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{1}{m} \int_C x\rho(x, y) ds, \quad \bar{y} = \frac{1}{m} \int_C y\rho(x, y) ds.$$

Find the mass and center of mass of a wire bent in the shape of a semicircle $x^2 + y^2 = 4, x \geq 0$ with linear density $\rho(x, y) = k$ where k is a constant.