Math 6A Practice Problems III

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1. Use the transformation $x = 2u + v, y = u + 2v$ to evaluate the integral $\iint_R (x - 3y)dA$, where R is the triangular region with vertices $(0, 0), (2, 1), (1, 2)$.

2. Evaluate the surface integral $\iint_S x^2yzdS$ where S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $[0, 3] \times [0, 2]$.

3. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ and S is the boundary of the solid region E enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.

4. Use Stokes' Theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = x^2z^2\mathbf{i} + y^2z^2\mathbf{j} + xyz\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$.

5. Consider the vector field $\mathbf{F}(x, y, z) = yz\mathbf{i} + 2xz\mathbf{j} + e^{xy}\mathbf{k}$, where C is circle $x^2 + y^2 = 16, z = 5$ oriented counterclockwise when viewed from above.
   (a) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ by finding an appropriate parametrization vector $\mathbf{r}(t)$.
   (b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Stokes' Theorem, and verify it is equal to your solution in part (a).

6. Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$ where E is the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
   Note: to verify the theorem is true you need to show that $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F}dV$; that is, you need to calculate both integrals and show they are equal.

7. Use the Divergence Theorem to calculate the surface integral $\iint \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of $\mathbf{F}$ across S where $\mathbf{F}(x, y, z) = (\cos z + xy^2)\mathbf{i} + xe^{-z}\mathbf{j} + (\sin y + x^2z)\mathbf{k}$, and S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.