Math 6B Practice Problems II

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1. Find the radius and interval convergence of the series:
   (a) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$
   (b) $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$
   (c) $\sum_{n=1}^{\infty} n!(2x-1)^n$

2. Use the power series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ to find the power series of the following functions:
   (a) $f(x) = \frac{1}{x^7 - 1}$
   (b) $f(x) = \frac{x^3}{x+2}$

3. Find the Taylor series for $f(x) = \frac{1}{x}$ centered at $a = -3$.

4. Evaluate the indefinite integral as an infinite series:
   (a) $\int e^{x^2} dx$
   (b) $\int x \cos(x^3) dx$

5. Use series to evaluate the limit $\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$.

6. Find the sum of the series:
   (a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$
   (b) $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + ...$

7. Find the Fourier series of the function:
   (a) $f(x) = 1 - \sin x + 3\cos 2x + \sin^2(3x)$, $-\pi \leq x \leq \pi$
   (b) $f(x) = x^2$, $-p \leq x \leq p$
   (c) $f(x) = x \cos x$ if $-\frac{\pi}{2} < x < \frac{\pi}{2}$
   Hint: This is a tough one. Use the fact that the function is odd. Also use the formula $\cos(\alpha)\sin(\beta) = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$.

8. Find the cosine and sine series of the function $f(x) = x$, $0 < x < 1$.

9. Find the Fourier transforms of the following functions:

   (a) $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a. \end{cases}$

   (b) $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & x \leq 0. \end{cases}$

   (c) $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1, \\ 0, & \text{otherwise.} \end{cases}$

10. Solve the system $u_{tt} = 16u_{xx}, 0 \leq x \leq 1, t \geq 0; u(0,t) = u(1,t) = 0; u(x,0) = f(x), u_t(x,0) = 0$ where

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ 2(1-x), & \frac{1}{2} < x \leq 1. \end{cases}$$

11. Solve the system $u_{tt} = u_{xx}, u(x,0) = \sin \pi x + 3 \sin 2\pi x, u_t(x,0) = \sin \pi x, -\infty < x < \infty, t \geq 0.$

12. Solve the system $u_t = u_{xx}, 0 \leq x \leq 1, t \geq 0; u(0,t) = u(1,t) = 0; u(x,0) = e^{-x}.$