Math 6B: PDE "Quiz"

May 23, 2016

Name: _____ Score: NA

Directions: Open book, open note, open neighbor.

Disclaimer: The content and level of difficulty of this quiz are not guaranteed to be in correlation with the midterm nor final examinations in any form.

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1. Vibration of a stretched string with fixed ends: The ends of a stretched string of length $L = 1$ are fixed at $x = 0$ and $x = 1$. The string is set to vibrate from rest by releasing it from an initial triangular shape model by the function

$$f(x) = \begin{cases} \frac{3}{10}x, & 0 \le x \le \frac{1}{3} \\ \frac{3(1-x)}{20}, & \frac{1}{3} \le x \le 1. \end{cases}$$

   Determine the subsequent motion of the string (find $u(x,t)$) given that $c = 1/\pi$.

2. A thin bar of length π units is placed in boiling water (temperature 100°C). After reaching 100°C throughout, the bar is removed from the boiling water. With the lateral sides kept insulated, suddenly, at time $t = 0$, the ends are immersed in a medium with constant freezing temperature 0°C. Taking $c = 1$, find the temperature $u(x, t)$ for $t > 0$.

   Hint: You need to solve the system $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}, u(0, t) = u(\pi, t) = 0, u(x, 0) = 100, 0 < x < \pi, t > 0.$