

Math 6B Practice Problems II

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1. Find the radius and interval convergence of the series:

(a) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$

(c) $\sum_{n=1}^{\infty} n!(2x-1)^n$

2. Use the power series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ to find the power series of the following functions:

(a) $f(x) = \frac{1}{x^7 - 1}$

(b) $f(x) = \frac{x^3}{x+2}$

3. Find the Taylor series for $f(x) = \frac{1}{x}$ centered at $a = -3$.

4. Evaluate the indefinite integral as an infinite series:

(a) $\int e^{x^2} dx$

(b) $\int x \cos(x^3) dx$

5. Use series to evaluate the limit $\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3}$.

6. Find the sum of the series:

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

(b) $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$

7. Find the Fourier series of the function:

(a) $f(x) = 1 - \sin x + 3 \cos 2x + \sin^2(3x)$, $-\pi \leq x \leq \pi$

(b) $f(x) = x^2$, $-p \leq x \leq p$

(c) $f(x) = x \cos x$ if $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Hint: This is a tough one. Use the fact that the function is odd. Also use the formula $\cos(\alpha) \sin(\beta) = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$.

8. Find the cosine and sine series of the function $f(x) = x$, $0 < x < 1$.

9. Solve the system $u_{tt} = 16u_{xx}$, $0 \leq x \leq 1$, $t \geq 0$; $u(0, t) = u(1, t) = 0$; $u(x, 0) = f(x)$, $u_t(x, 0) = 0$ where

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ 2(1-x), & \frac{1}{2} < x \leq 1. \end{cases}$$

10. Solve the system $u_t = u_{xx}$, $0 \leq x \leq 1$, $t \geq 0$; $u(0, t) = u(1, t) = 0$; $u(x, 0) = e^{-x}$.