Math 1 – Worksheet 1

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Name: KEY

1. Consider the function $f(x) = x^2 - x$.

   (a) Evaluate $f(0)$, $f(1)$. Is f a one-to-one function?

   $f(0) = 0^2 - 0 = 0 - 0 = 0$ $\boxed{f(0)=0}$

   $f(1) = 1^2 - 1 = 1 - 1 = 0$ $\boxed{f(1)=0}$

   f is not one-to-one, there are two inputs that have the same output

   (b) Find $f(a)$ and $f(a+h)$.

   $\boxed{f(a) = a^2 - a}$

   $f(a+h) = (a+h)^2 - (a+h) = (a+h)(a+h) - a - h = a^2 + 2ah + h^2 - a - h$

   $\boxed{f(a+h) = a^2 + 2ah + h^2 - a - h}$

   (c) Use your work in (b) to evaluate $\dfrac{f(a+h) - f(a)}{h}$. Simplify as much as possible. This expression is called the rate of change of $f(x)$ on the interval $[a, a+h]$.

   $$\frac{f(a+h) - f(a)}{h} = \frac{a^2 + 2ah + h^2 - a - h - (a^2 - a)}{h} = \frac{\cancel{a^2} + 2ah + h^2 - \cancel{a} - h - \cancel{a^2} + \cancel{a}}{h}$$

   $$= \frac{2ah + h^2 - h}{h} = \frac{\cancel{h}(2a + h - 1)}{\cancel{h}} = \boxed{2a + h - 1}$$

* use vertical line test to test if function

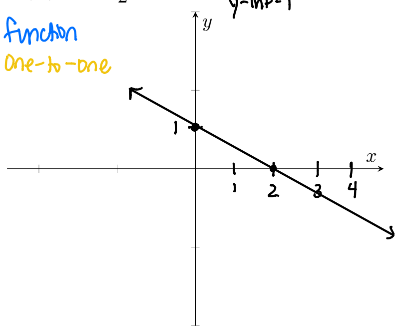
* use horizontal line test to test for one-to-one function

2. Use the following plots to sketch graphs of the equations (a) $f(x) = -\frac{1}{2}x + 1$, (b) $y = x^2$, (c) $y = \sqrt{x}$, (d) $x^2 + y^2 = 4$. Which of these expressions are functions? Which are 1−1 functions?

(a) $f(x) = -\frac{1}{2}x + 1$ slope = $-1/2$ y-int = 1

function

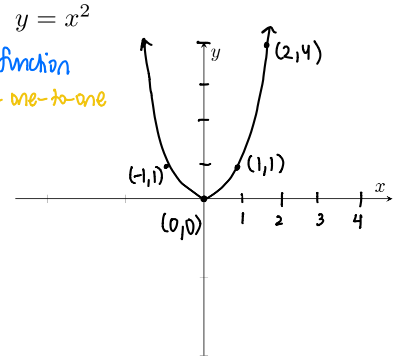
one-to-one



(b) $y = x^2$

function

not one-to-one



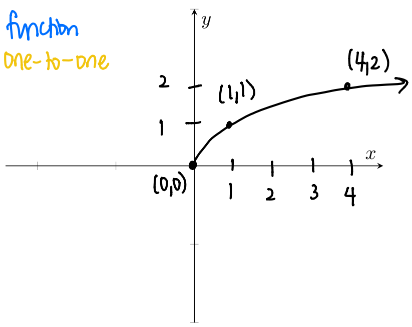
x	$y=x^2$
1	1
0	0
-1	1
2	4

(c) $y = \sqrt{x}$

function

one-to-one

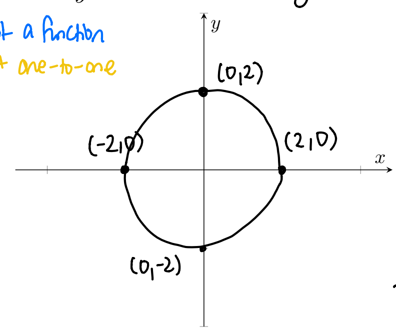
x	$y=\sqrt{x}$
0	0
1	1
4	2
-1	undefined



(d) $x^2 + y^2 = 4$ $\longrightarrow$ $y^2 = 4 - x^2 \rightarrow y = \pm\sqrt{4-x^2}$

not a function

not one-to-one



x	y
0	±2
±2	0

* Note: $x^2 + y^2 = R^2$ is equation of circle w/ center (0,0), radius R

3. Write the domain and ranges of the following equations (*Hint*: use your sketches in the previous exercise). Use both interval and set builder notation.

(a) $f(x) = -\frac{1}{2}x + 1$

Domain: $(-\infty, \infty)$ or $\{x : x \text{ is a real number}\}$

Range: $(-\infty, \infty)$ or $\{x : x \text{ is a real number}\}$

(b) $y = x^2$

Domain: $(-\infty, \infty)$ or $\{x : x \text{ is a real number}\}$

Range: $[0, \infty)$ or $\{y : y \geq 0\}$

(c) $y = \sqrt{x}$

Domain: $[0, \infty)$ or $\{x : x \geq 0\}$

Range: $[0, \infty)$ or $\{y : y \geq 0\}$

(d) $x^2 + y^2 = 4$

Domain: $[-2, 2]$ or $\{x : -2 \leq x \leq 2\}$

Range: $[-2, 2]$ or $\{y : -2 \leq y \leq 2\}$

4. Find the domain of the following functions. Use both interval and set builder notation.

(a) $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

denominator $\Rightarrow$ $x^2+1 = 0$
square root $\Rightarrow$ $x^2+1 \geq 0$
both say $x^2+1>0$

But x^2+1 is always >0 !

So domain is $\boxed{\mathbb{R} \text{ or } (-\infty, \infty)}$

(b) $g(x) = \dfrac{x}{\sqrt{x+1}}$

denominator $\Rightarrow$ $x+1 \neq 0$
square root $\Rightarrow$ $x+1 \geq 0$
both say $x+1>0$
$x>-1$

$\boxed{\{x: x>-1\}}$

-1

$(-1, \infty)$

(c) $h(x) = \sqrt[4]{9-x^2}$

even root $\Rightarrow$ $9-x^2 \geq 0$
$(3-x)(3+x) \geq 0$

Find where $=0$: $3-x=0$, $x=3$; $3+x=0$, $x=-3$

-3 3

Test: if $x=-4$, is $9-(-4)^2 \geq 0$? no.
if $x=0$, is $9-0^2 \geq 0$? yes, shade the region (blue).
if $x=4$, is $9-4^2 \geq 0$? no.

$\boxed{\{x: -3 \leq x \leq 3\}, \ [-3,3]}$

(d) $k(x) = \dfrac{1}{x^2-x} = \dfrac{1}{x(x-1)}$

Don't want denominator $=0$.

$x(x-1) \neq 0$
$x \neq 0 \quad x \neq 1$

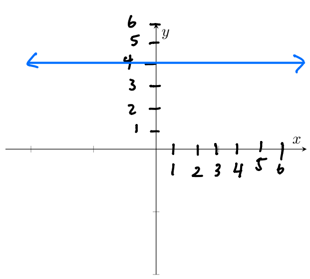
$\boxed{\{x: x\neq 0, x \neq 1\}}$

0 1

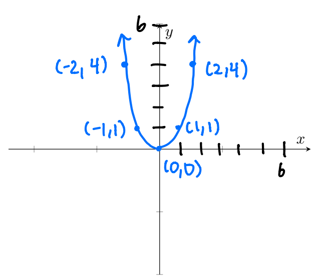
$\boxed{(-\infty, 0) \cup (0,1) \cup (1, \infty)}$

5. In this exercise we will practice graphing piecewise functions.

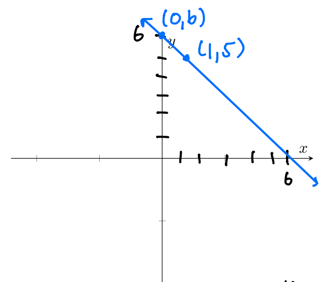
(a) Graph the functions $f(x) = 4$, $f(x) = x^2$, $f(x) = -x+6$ on the following graphs.



$f(x)=4$



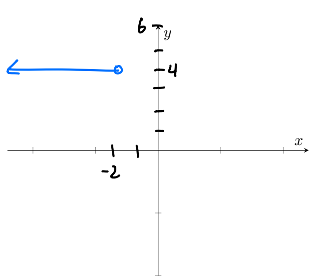
$f(x)=x^2$



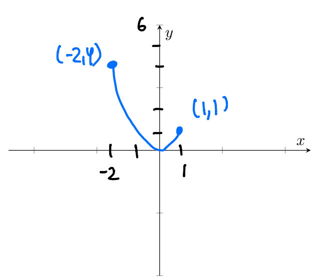
$f(x) = -x+6$

line w/ slope -1
y-int 6

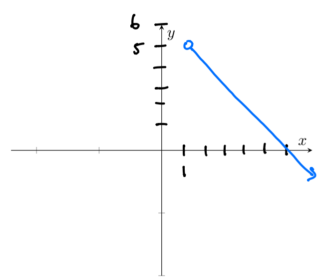
(b) Using your previous plots, plot $f(x) = 4$ for $x < -2$ on the first graph, $f(x) = x^2$ for $-2 \leq x \leq 1$ on the second graph, $f(x) = -x+6$ for $x > 1$ on the third graph.



$x<-2$ means left of -2
w/ open circle when $x=-2$

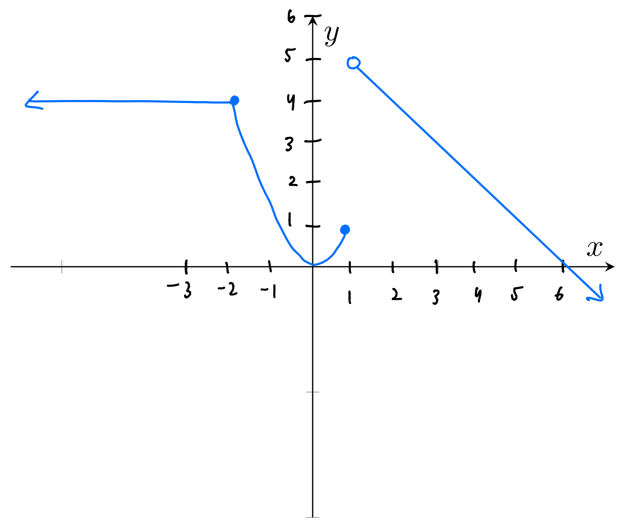


$-2 \leq x \leq 1$ means right of -2
left of 1
closed circles when $x=-2, 1$



$x>1$ means right of 1
w/ open circle when $x=-1$

(c) Now plot the graph of $f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \le x \le 1 \\ -x+6, & x > 1 \end{cases}$.



(d) What is the domain and range of this function?

Domain : $(-\infty, \infty)$ or $\{x : x \text{ is a real number}\}$

Range: $(-\infty, 5)$

(e) Is this function one-to-one?

no, does not pass horizontal line test