Week 2 Solutions

Math 31AL

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Tuesday, October 8

1. (a) $f(-4) = 3$, $\lim_{x\to -4^+} f(x) = -2$, $\lim_{x\to -4^-} f(x) = 3$, $\lim_{x\to -4} f(x) = \text{DNE}$

   (b) $f(-1) = 4$, $\lim_{x\to -1^+} f(x) = 4$, $\lim_{x\to -1^-} f(x) = 4$, $\lim_{x\to -1} f(x) = 4$

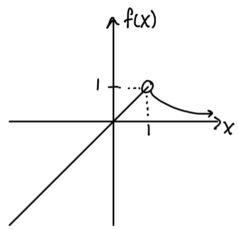
   (c) $f(2) = -1$, $\lim_{x\to 2^+} f(x) = 5$, $\lim_{x\to 2^-} f(x) = -1$, $\lim_{x\to 2} f(x) = \text{DNE}$

   (d) $f(4) =$ undefined, $\lim_{x\to 4^+} f(x) = 2$, $\lim_{x\to 4^-} f(x) = 2$, $\lim_{x\to 4} f(x) = 2$

2. $\lim_{x\to -1^-} g(x) = \infty$, $\lim_{x\to -1^+} g(x) = -\infty$, $\lim_{x\to -1} g(x) = \text{DNE}$

   $\lim_{x\to 2^-} g(x) = -\infty$, $\lim_{x\to 2^+} g(x) = -\infty$, $\lim_{x\to 2} g(x) = -\infty$

3. $f(x) = \begin{cases} \frac{1}{x}, & x > 1 \\ x, & x < 1 \end{cases}$



   $\lim_{x\to 1^-} f(x) = 1$ $\quad \lim_{x\to 1^+} f(x) = 1 \rightarrow \lim_{x\to 1} f(x) = 1$

   not continuous at 1 since $f(1)$ is undefined

4. Need limit to exist for $g(x)$ to be continuous

   $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} 2x + c = 2 + c$

   $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} x^2 + 3 = 1 + 3 = 4$

   $2 + c = 4 \Rightarrow c = 2$

5. $f(x) = x^2$

   $f(4) = 4^2 = 16$

   $f(p) = p^2$

   $f(x+h) = (x+h)^2$

   $$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{(x+h)(x+h) - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h} = 2x + h$$

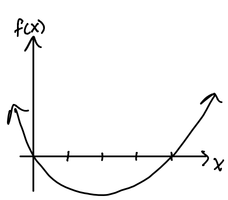
6. $$\frac{\frac{t}{t}\cdot\frac{1}{t+1} - \frac{1}{t}\cdot\frac{t+1}{t+1}}{\frac{t}{t}\cdot\frac{1}{t+1} + \frac{1}{t}\cdot\frac{t+1}{t+1}} = \frac{\frac{t}{t(t+1)} - \frac{t+1}{t(t+1)}}{\frac{t}{t(t+1)} + \frac{t+1}{t(t+1)}} = \frac{\frac{t-(t+1)}{t(t+1)}}{\frac{t+t+1}{t(t+1)}} = \frac{\frac{-1}{t(t+1)}}{\frac{2t+1}{t(t+1)}} = \frac{-1}{\cancel{t(t+1)}}\cdot\frac{\cancel{t(t+1)}}{2t+1} = \frac{-1}{2t+1}$$

7. $f(x) = x^2 - 4x$

   $x^2 - 4x = 0$

   $x(x-4) = 0$

   $x = 0,\ x = 4$



   $$\text{Ave. rate of change} = \frac{f(5) - f(t)}{5-t} = \frac{5^2 - 4(5) - (t^2 - 4t)}{5-t} = \frac{5 - t^2 + 4t}{5-t} = \frac{-(t^2 - 4t - 5)}{5-t}$$

   $$= \frac{\cancel{-}(\cancel{t-5})(t+1)}{\cancel{5-t}} = t+1$$

   $$\lim_{t\to 5} \frac{f(5) - f(t)}{5-t} = \lim_{t\to 5} t+1 = 5+1 = 6$$

Thursday, October 10

① (a) $$\lim_{x\to\infty} \frac{2x^2+3x}{x^2+x-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{3x}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{2+\frac{3}{x}}{1+\frac{1}{x}-\frac{1}{x^2}} = \frac{2+0}{1+0-0} = 2$$

(b) $$\lim_{x\to\infty} \frac{2x^2+3x}{x^3+x-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{3x}{x^2}}{\frac{x^3}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{2+\frac{3}{x}}{x+\frac{1}{x}-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{2+0}{x+0-0} = \lim_{x\to\infty}\frac{2}{x} = 0$$

$\uparrow$

* You may have picked a different ratio and that is ok!

(c) $$\lim_{x\to\infty} \frac{2x^5+3x}{x^2+x-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{2x^5}{x^2}+\frac{3x}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{2x^3+\frac{3}{x}}{1+\frac{1}{x}-\frac{1}{x^2}} = \frac{\infty+0}{1+0-0} = \infty$$

Observation: * If deg top = deg bottom, limit is ratio of leading coefficients
* If deg top < deg bottom, limit is 0
* If deg top > deg bottom, limit is ∞

② $$\sqrt{x^2} = |x| = \begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0\end{cases} \implies \frac{1}{x} = \begin{cases} \sqrt{\frac{1}{x^2}} & \text{if } x\geq 0 \\ -\sqrt{\frac{1}{x^2}} & \text{if } x<0\end{cases}$$

$$\lim_{x\to\infty}\frac{2x-3}{\sqrt{9x^2+x+1}}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{2-\frac{3}{x}}{\sqrt{9x^2+x+1}\,\sqrt{\frac{1}{x^2}}} = \lim_{x\to\infty}\frac{2-\frac{3}{x}}{\sqrt{\frac{9x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}} = \lim_{x\to\infty}\frac{2-\frac{3}{x}}{\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}} = \frac{2-0}{\sqrt{9+0+0}} = \frac{2}{3}$$

$$\lim_{x\to-\infty}\frac{2x-3}{\sqrt{9x^2+x+1}}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{2-\frac{3}{x}}{\sqrt{9x^2+x+1}\left(-\sqrt{\frac{1}{x^2}}\right)} = \lim_{x\to-\infty}\frac{2-\frac{3}{x}}{-\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}} = \frac{2}{-\sqrt{9}} = -\frac{2}{3}$$

③ (a) $m^2-n^2 = \boxed{(m-n)(m+n)}$

(b) $x^2-36 = \boxed{(x-6)(x+6)}$

(c) $4^{2t}-1 = (4^t)^2-1 = \boxed{(4^t-1)(4^t+1)}$

(d) $y^3-27 = \boxed{(y-3)(y^2+3y+9)}$

(e) $t^4-16 = (t^2-4)(t^2+4) = \boxed{(t-2)(t+2)(t^2+4)}$

④ $$\lim_{x\to5}\frac{2x^2-9x-5}{x^2-25} = \frac{2(5)^2-9(5)-5}{5^2-25} = \frac{50-45-5}{0} = \frac{0}{0}$$ ☹

Factor,

$2x^2-9x-5$ $\quad -10 <{}^{-10}_{\ 1}$

$2x^2-10x+x-5$

$2x(x-5)+1(x-5)$

$(2x+1)(x-5)$

x^2-25

$(x-5)(x+5)$

$$\lim_{x\to5}\frac{2x^2-9x-5}{x^2-25} = \lim_{x\to5}\frac{(2x+1)\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} = \lim_{x\to5}\frac{2x+1}{x+5} = \frac{2(5)+1}{5+5} = \frac{11}{10}$$

⑤ $$\lim_{x\to16}\frac{\sqrt{x}-4}{x-16}$$

One way: $$\lim_{x\to16}\frac{\sqrt{x}-4}{x-16} = \lim_{x\to16}\frac{\cancel{\sqrt{x}-4}}{\underbrace{\cancel{(\sqrt{x}-4)}(\sqrt{x}+4)}_{\text{diff. of squares}}} = \lim_{x\to16}\frac{1}{\sqrt{x}+4} = \frac{1}{4+4} = \frac{1}{8}$$

Another way: $$\lim_{x\to16}\frac{\sqrt{x}-4}{x-16}\cdot\underbrace{\frac{(\sqrt{x}+4)}{(\sqrt{x}+4)}}_{\text{conjugate}} = \lim_{x\to16}\frac{\cancel{x-16}}{\cancel{(x-16)}(\sqrt{x}+4)} = \lim_{x\to16}\frac{1}{\sqrt{x}+4} = \frac{1}{4+4} = \frac{1}{8}$$