Week 3 Solutions

Math 31AL
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Tuesday, October 15

① (a) $\lim_{y\to 3} \frac{y^2+y-12}{y^4-8y^2-5y+6} = \frac{3^2+3-12}{3^4-8(3)^2-5(3)+6} = \frac{9+3-12}{81-8(9)-15+6} = \frac{0}{81-72-15+6} = \frac{0}{0}$ ☹

3 is a zero of y^2+y-12 & $y^4-8y^2-5y+6 \Rightarrow (y-3)$ is a factor of y^2+y-12 and y^4-8y^2-5y+6

(b) Factor: $y^2+y-12 = (y+4)(y-3)$

Long division:

$$\begin{array}{r|l} & y^3+3y^2+y-2 \\ y-3 & y^4+0y^3-8y^2-5y+6 \\ & -(\cancel{y^4}-3y^3) \\ & \quad 3y^3-8y^2 \\ & \quad -(\cancel{3y^3}-9y^2) \\ & \qquad y^2-5y \\ & \qquad -(\cancel{y^2}-3y) \\ & \qquad\quad -2y+6 \\ & \qquad\quad -(-\cancel{2y}+6) \\ & \qquad\qquad 0 \checkmark \end{array}$$

So $y^4-8y^2-5y+6 = (y-3)(y^3+3y^2+y-2)$

(c) $\lim_{y\to 3} \frac{y^2+y-12}{y^4-8y^2-5y+6} = \lim_{y\to 3} \frac{\cancel{(y-3)}(y+4)}{\cancel{(y-3)}(y^3+3y^2+y-2)} = \lim_{y\to 3} \frac{y+4}{y^3+3y^2+y-2} = \frac{3+4}{3^3+3(3)^2+3-2} = \frac{7}{27+27+1} = \frac{7}{55}$

② (a) $\frac{\frac{4}{4}\frac{1}{(h+2)^2} - \frac{1}{4}\frac{(h+2)^2}{(h+2)^2}}{h} = \frac{\frac{4}{4(h+2)^2} - \frac{(h+2)^2}{4(h+2)^2}}{h} = \frac{\frac{4-(h+2)^2}{4(h+2)^2}}{\frac{h}{1}} = \frac{4-(h^2+4h+4)}{4(h+2)^2}\frac{1}{h} = \frac{\cancel{4}-h^2-4h\cancel{-4}}{4h(h+2)^2} = \frac{-\cancel{h}(h+4)}{4\cancel{h}(h+2)^2} = \frac{-(h+4)}{4(h+2)^2}$

(b) $\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x^2+x}} = \frac{\sqrt{x+1}}{\sqrt{x+1}}\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x(x+1)}} = \boxed{\frac{\sqrt{x+1}-1}{\sqrt{x(x+1)}}}$

(c) $\frac{1}{\sqrt{x}-2} - \frac{4}{x-4}$

one way conjugate $\frac{\sqrt{x}+2}{\sqrt{x}+2}\frac{1}{\sqrt{x}-2} - \frac{4}{x-4} = \frac{\sqrt{x}+2-4}{(\sqrt{x}+2)(\sqrt{x}-2)} = \frac{\cancel{\sqrt{x}-2}}{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}} = \frac{1}{\sqrt{x}+2}$

another way factor $x-4$ first $\frac{\sqrt{x}+2}{\sqrt{x}+2}\frac{1}{\sqrt{x}-2} - \frac{4}{(\sqrt{x}-2)(\sqrt{x}+2)} = \frac{\cancel{\sqrt{x}-2}}{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}} = \frac{1}{\sqrt{x}+2}$

Thursday, Oct 17

① $\lim_{x\to 0} \frac{\sin x}{x} = 1$, $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

② $\lim_{x\to 0} \frac{\sin 7x}{3x}$ (a) $u = 7x$ (b) $x = \frac{u}{7}$ (c) if $u\to 0$ then $x = \frac{u}{7} \to \frac{0}{7} = 0$

(d) $\lim_{x\to 0} \frac{\sin 7x}{3x} = \lim_{u\to 0} \frac{\sin u}{3\cdot\frac{u}{7}}$ (e) $\lim_{u\to 0} \frac{\sin u}{\frac{3}{7}u} = \frac{1}{\frac{3}{7}} \lim_{u\to 0} \cancel{\frac{\sin u}{u}}^{1} = \frac{7}{3}$

③ (a) $\lim_{x\to 0} \frac{\sin^2 x}{x^2} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} = \cancel{\lim_{x\to 0} \frac{\sin x}{x}}^{1} \cancel{\lim_{x\to 0} \frac{\sin x}{x}}^{1} = 1 \cdot 1 = 1$

(b) Since $\lim_{x\to 0} \frac{\sin x}{x} = 1$, then $\lim_{x\to 0} \frac{x}{\sin x} = \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}} = \frac{1}{1} = 1$, therefore $\lim_{x\to 0} \frac{x^2}{\sin^2 x} = \lim_{x\to 0} \frac{x \cdot x}{\sin x \cdot \sin x} = \cancel{\lim_{x\to 0} \frac{x}{\sin x}}^{1} \cancel{\lim_{x\to 0} \frac{x}{\sin x}}^{1} = 1$

④ $\lim_{t\to 0} \frac{\tan 4t}{t \sec t} = \lim_{t\to 0} \frac{\frac{\sin 4t}{\cos 4t}}{t \cdot \frac{1}{\cos t}} = \lim_{t\to 0} \frac{\frac{\sin 4t}{\cos 4t}}{\frac{t}{\cos t}} = \lim_{t\to 0} \frac{\sin 4t}{\cos 4t} \frac{\cos t}{t} = \lim_{t\to 0} \frac{\sin 4t}{t} \lim_{t\to 0} \frac{\cos t}{\cos 4t} = \lim_{u\to 0} \frac{\sin u}{\frac{u}{4}} \lim_{t\to 0} \frac{\cos t}{\cos 4t}$

$u = 4t$
$t = u/4$

$= \frac{1}{\frac{1}{4}} \cancel{\lim_{u\to 0} \frac{\sin u}{u}}^{1} \lim_{t\to 0} \frac{\cos t}{\cos 4t} = 4 \cdot 1 \cdot \frac{\cos 0}{\cos 0} = 4 \cdot 1 \cdot \frac{1}{1} = 4$

⑤ (a) $\lim_{h\to 0} \frac{\frac{2}{2}\frac{1}{2+h} - \frac{1}{2}\frac{2+h}{2+h}}{h} = \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{\cancel{2}-\cancel{2}-\cancel{h}}{2(2+h)} \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-1}{2(2+h)} = -\frac{1}{4}$

(b) $\lim_{h\to 0} \frac{\frac{\sqrt{4}}{\sqrt{4}}\frac{1}{\sqrt{4+h}} - \frac{1}{\sqrt{4}}\frac{\sqrt{4+h}}{\sqrt{4+h}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{4}-\sqrt{4+h}}{\sqrt{4}\sqrt{4+h}}}{\frac{h}{1}} = \lim_{h\to 0} \frac{\sqrt{4}-\sqrt{4+h}}{\sqrt{4}\sqrt{4+h}} \frac{1}{h} \frac{\sqrt{4}+\sqrt{4+h}}{\sqrt{4}+\sqrt{4+h}} = \lim_{h\to 0} \frac{4-(4+h)}{\sqrt{4}h\sqrt{4+h}} = \lim_{h\to 0} \frac{\cancel{4}-\cancel{4}-\cancel{h}}{\sqrt{4}\cancel{h}\sqrt{4+h}}$

$= -\lim_{h\to 0} \frac{-1}{\sqrt{4}\sqrt{4+h}} = \frac{-1}{\sqrt{4}\sqrt{4}} = \frac{-1}{2(2)} = -\frac{1}{4}$

(c) $\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$f(a) = \frac{1}{2}$
$f(a+h) = \frac{1}{2+h}$

looks like $a=2$, $f(x) = \frac{1}{x}$
$\Rightarrow$ $\boxed{f'(2) \text{ for } f(x) = \frac{1}{x}}$

$\lim_{h\to 0} \frac{\frac{1}{\sqrt{4+h}} - \frac{1}{\sqrt{4}}}{h} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$f(a) = \frac{-1}{\sqrt{4}}$
$f(a+h) = \frac{1}{\sqrt{4+h}}$

looks like $a=4$, $f(x) = \frac{1}{\sqrt{x}}$
$\Rightarrow$ $\boxed{f'(4) \text{ for } f(x) = \frac{1}{\sqrt{x}}}$