# Week 5 Solutions

Math 31AL
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Tuesday, October 29

① $y = x^{-1}$

$y' = -1x^{-2}$ (leave unsimplified to find the pattern)

$y'' = (-1)(-2)x^{-3} = (-1)^2\, 1\cdot 2 x^{-3} = (-1)^2 2!\, x^{-3}$

$y''' = (-1)(-2)(-3)x^{-4} = (-1)^3\, 1\cdot 2\cdot 3\, x^{-4} = (-1)^3\, 3!\, x^{-4}$

$y^{(4)} = (-1)(-2)(-3)(-4)x^{-5} = (-1)^4\, 1\cdot 2\cdot 3\cdot 4\, x^{-5} = (-1)^4\, 4!\, x^{-5}$

$y^{(n)} = (-1)^n\, n!\, x^{-(n+1)}$

② $xy + x^2y^2 = 6$

$(xy)' + (x^2y^2)' = 6'$

$(x)'y + x(y)' + (x^2)'y^2 + x^2(y^2)' = 0$

$1\cdot y + x\cdot 1\cdot y' + 2x\, y^2 + x^2\cdot 2y\cdot y' = 0$

$y + xy' + 2xy^2 + 2x^2yy' = 0$

$xy' + 2x^2yy' = -y - 2xy^2$

$y'(x + 2x^2y) = -y - 2xy^2$

$y' = \dfrac{-y-2xy^2}{x+2x^2y}$

Plug in (2,1) to get the slope:

$m = \dfrac{-1-2(2)(1)^2}{2+2(2)^2(1)} = \dfrac{-1-4}{2+8} = \dfrac{-5}{10} = \dfrac{-1}{2}$

Equation of line: $y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{2}(x-2)$

$y - 1 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 2$

③ (a) $y = \sin^3(4x) = (\sin(4x))^3$ $\boxed{y' = 3(\sin(4x))^2\cdot\cos(4x)\cdot 4}$

(b) $y = (3x-2x^2)^4$ $\boxed{y' = 4(3x-2x^2)^3\cdot(3-4x)}$

(c) $y = \left(\dfrac{3x-1}{x^2+3}\right)^2$ $y' = 2\left(\dfrac{3x-1}{x^2+3}\right)^1\cdot\left(\dfrac{(x^2+3)(3)-(3x-1)(2x)}{(x^2+3)^2}\right) = 2\left(\dfrac{3x-1}{x^2+3}\right)\left(\dfrac{3x^2+9-6x^2+2x}{(x^2+3)^2}\right)$

$= \boxed{\dfrac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}}$

(d) $f(x) = \sin\left(\frac{1}{x^2}\right) = \sin(x^{-2})$ $f'(x) = \cos(x^{-2})\cdot -2x^{-3} = \boxed{\dfrac{-2\cos\left(\frac{1}{x^2}\right)}{x^3}}$

③ continued

(e) $y = \dfrac{4}{(3x^2-2x+1)^3} = 4(3x^2-2x+1)^{-3}$

$y' = 4 \cdot -3(3x^2-2x+1)^{-4}(6x-2)$

$= \boxed{\dfrac{-12(6x-2)}{(3x^2-2x+1)^4}}$

## Thursday, October 31

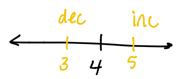
① $f(x) = x^3 - 6x^2 + 8$, Find absolute extrema on $[1,6]$

$f'(x) = 3x^2 - 12x = 0$

$3x(x-4) = 0$

$\cancel{x=0}$ $\quad x = 4$

not in $[1,6]$ so do not need to test




$f'(3) = 3(3)^2 - 12(3) = 27-36 < 0$

$f'(5) = 3(5)^2 - 12(5) = 75 - 60 > 0$

since dec → inc, local min at $x=4$

Test endpoints & critical points:

$f(4) = 4^3 - 6(4)^2 + 8 = 64 - 96 + 8 = -24$ ← $\boxed{(4,-24) \text{ absolute min}}$

$f(1) = 1^3 - 4(1)^2 + 8 = 1 - 4 + 8 = 5$

$f(6) = 6^3 - 6(6^2) + 8 = \cancel{6^3} - \cancel{6^3} + 8 = 8$ ← $\boxed{(6,8) \text{ absolute max}}$

② $f(x) = \dfrac{x}{x^2+5}$, Find absolute extrema on $[4,8]$

$f'(x) = \dfrac{(x^2+5)x' - x(x^2+5)'}{(x^2+5)^2} = \dfrac{x^2+5-x(2x)}{(x^2+5)^2} = \dfrac{x^2+5-2x^2}{(x^2+5)^2} = \dfrac{-x^2+5}{(x^2+5)^2}$

set numerator & denominator $= 0$

$-x^2+5 = 0$ $\qquad (x^2+5)^2 = 0$

$x^2 = 5$ $\qquad x^2+5 = 0$

$\cancel{x = \pm\sqrt{5}}$ $\qquad x^2 = -5$

no real solution

not in $[4,8]$ so do not need to test

Test endpoints:

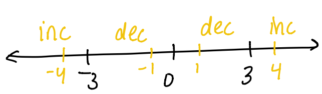
$f(4) = \dfrac{4}{4^2+5} = \dfrac{4}{21} = 0.19047\ldots$ ← absolute max at $\left(4, \frac{4}{21}\right)$

$f(8) = \dfrac{8}{8^2+5} = \dfrac{8}{69} = 0.115942\ldots$ ← absolute min at $\left(8, \frac{8}{69}\right)$

③ $f(x) = \dfrac{x^5}{5} - 3x^3 + 6$, Find local extrema

$f'(x) = \dfrac{5x^4}{5} - 9x^2 = x^4 - 9x^2 = x^2(x^2-9) = x^2(x-3)(x+3) = 0$

$x=0,\ x=3,\ x=-3$




Going to use $f'(x) = x^2(x-3)(x+3)$

$f'(-4) = (-4)^2(-4-3)(-4+3) = 16(-7)(-1) > 0$

$f'(-1) = (-1)^2(-1-3)(-1+3) = 1(-4)(2) < 0$

$f'(1) = 1^2(1-3)(1+3) = 1(-2)(4) < 0$

$f'(4) = 4^2(4-3)(4+3) = 16(1)(7) > 0$

local max means inc→dec

local min means dec→inc

| |
|---|
| local max at $x=-4$ |
| local min at $x=4$ |
| neither at $x=0$ |

④ Implicit differentiation at (1,1) $\left(y^4+xy\right)'=\left(2x^3-7x+7\right)'$

$$4y^3y'+(x)'y+x(y)'=6x^2-7$$

$$4y^3y'+y+xy'=6x^2-7$$

$$4y^3y'+xy'=6x^2-7-y$$

$$y'(4y^3+x)=6x^2-7-y$$

$$y'=\frac{6x^2-7-y}{4y^3+x} \quad \text{so} \quad m=\frac{6\cdot1^2-7-1}{4\cdot1^3+1}=\frac{6-7-1}{4+1}=\frac{-2}{5}$$

Tangent line: $y-1=\frac{-2}{5}(x-1)$

$$y=-\frac{2}{5}x+\frac{2}{5}+1$$

$$y=\frac{-2}{5}x+\frac{7}{5}$$

$$5y=-2x+7$$

$$2x+5y=7 \checkmark$$