

Math 33B

More Separable Equations Examples

Written by Victoria Kala, vткаla@math.ucla.edu

Thursday October 3, 2019

Ex Solve $\frac{dy}{dt} + 2ty^2 = 0$

① $\frac{dy}{dt} = -2ty^2$

$y^{-2} \frac{dy}{dt} = -2t$

② $\int y^{-2} \frac{dy}{dt} dt = \int -2t dt$

$\frac{y^{-1}}{-1} = -t^2 + C$

③ $y^{-1} = t^2 + C$
 $y = \frac{1}{t^2 + C}$

Ex Solve $(e^y - y) \cos t \frac{dy}{dt} = e^y \sin 2t$, $y(0) = 0$

Hints: • integration by parts

• $\sin 2t = 2 \sin t \cos t$

• implicit solution

① $\left(\frac{e^y - y}{e^y} \right) \frac{dy}{dt} = \frac{\sin 2t}{\cos t}$

$\left(\frac{e^y}{e^y} - \frac{y}{e^y} \right) \frac{dy}{dt} = \frac{2 \sin t \cos t}{\cos t}$

$(e^y - ye^{-y}) \frac{dy}{dt} = 2 \sin t$

② $\int (e^y - ye^{-y}) \frac{dy}{dt} dt = \int 2 \sin t dt$

$\int e^y \frac{dy}{dt} dt - \int ye^{-y} \frac{dy}{dt} dt = \int 2 \sin t dt$

$u = y \quad dv = e^{-y} \frac{dy}{dt} dt$
 $du = \frac{dy}{dt} dt \quad v = -e^{-y}$

$e^y - \left(-ye^{-y} + \int e^{-y} \frac{dy}{dt} dt \right) = -2 \cos t + C$

$e^y - (-ye^{-y} - e^{-y}) = -2 \cos t + C$

$e^y + ye^{-y} + e^{-y} = -2 \cos t + C$ ← * Note: too complicated to solve for y , so we leave as implicit solution

③ $y(0) = 0$ means $y = 0$ when $t = 0$. Plug in and solve for C :

$e^0 + 0e^{-0} + e^{-0} = -2 \cos 0 + C$

$1 + 0 + 1 = -2 + C$

$2 = -2 + C$

$C = 4$

Plug in C :

$e^y + ye^{-y} + e^{-y} = -2 \cos t + 4$