Math 31AL Practice Problems II

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1. Use chain rule to find the derivative of the following functions:

   (a) $f(x) = \cos^2(x^3 + 1)$ (b) $g(x) = \sqrt{x + \sin x}$

2. Find the equation of the tangent line at the given point using implicit differentiation:

$$2x^{1/2} + 4y^{-1/2} = xy, \quad (1, 4)$$

3. Let $f(x) = 2x^2 - 8x + 7$.

   (a) Find the local extrema of $f(x)$.

   (b) Find the absolute extrema of $f(x)$ on the interval $[0, 5]$.

   (c) Find the absolute extrema of $f(x)$ on the interval $[-4, 1]$.

4. Find all critical points of f and use the First or Second Derivative Test decide if they are local minima or maxima:

$$f(x) = \frac{x^2}{x + 1}$$

5. Let $f(x) = x^6 - 9x^4$.

   (a) Find the intercepts of f.

   (b) Find the critical points of f and intervals where f is increasing or decreasing.

   (c) Find the intervals where f is concave up or concave down. Does f have any inflection points?

   (d) Sketch the graph of $f(x)$.

6. Let $f(x) = \dfrac{x - 2}{x - 3}$.

   (a) Find the intercepts of f.

   (b) Find the vertical asymptotes of f.

   (c) Find the horizontal asymptotes of f.

   (d) Find the critical points of f and intervals where f is increasing or decreasing.

   (e) Find the intervals where f is concave up or concave down. Does f have any inflection points?

   (f) Sketch the graph of $f(x)$.

7. Victoria wishes to enclose a rectangular garden of area 1000m^2. One side will be enclosed with a brick wall costing \$90/m, the other three sides will be enclosed with a metal fence costing \$30/m. What dimensions of the garden minimize the total cost?

8. Let $f(x) = 9 - x^2$.

   (a) Sketch the graph of $f(x)$ on the interval $[0, 3]$.

   (b) Sketch the rectangles corresponding to L_3, then calculate L_3. Is this an underestimate or overestimate of the area under $f(x)$?

   (c) Sketch the rectangles corresponding to R_3, then calculate R_3. Is this an underestimate or overestimate of the area under $f(x)$?

   (d) Calculate a formula for R_N.

   (e) By taking the limit as $N \to \infty$ of your answer in (d), calculate the area under $f(x)$ on the interval $[0, 3]$.