Week 6 Example

Math 31AL
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Ex Sketch the graph of $f(x) = x(6-x)^{2/3}$ * Example from Pauli andré math notes

Intercepts: x-int, $0 = x(6-x)^{2/3}$ → $x=0$ or $(6-x)^{2/3}=0$, $6-x=0$, $x=6$

$(0,0)$ and $(6,0)$ are x-int

y-int, $f(0) = 0(6-0)^{2/3} = 0$

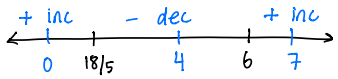
$(0,0)$ is y-int

1st derivative, use product & chain rule: $f'(x) = (x)'(6-x)^{2/3} + x\left((6-x)^{2/3}\right)' = 1\cdot(6-x)^{2/3} + x\cdot\frac{2}{3}(6-x)^{-1/3}\cdot(-1)$

$= \frac{3(6-x)^{1/3}}{3(6-x)^{1/3}}(6-x)^{2/3} - \frac{2x}{3(6-x)^{1/3}} = \frac{3(6-x)-2x}{3(6-x)^{1/3}} = \frac{18-3x-2x}{3(6-x)^{1/3}}$

$f'(x) = \frac{18-5x}{3(6-x)^{1/3}}$

Critical pts: $18-5x=0$, $5x=18$, $x=18/5$; $3(6-x)^{1/3}=0$, $(6-x)^{1/3}=0$, $6-x=0$, $x=6$



Test pts: $f'(0) = \frac{18-5(0)}{3(6-0)^{1/3}} = \frac{18}{3(6)^{1/3}} = \frac{+}{++} = +$

$f'(4) = \frac{18-5(4)}{3(6-4)^{1/3}} = \frac{18-20}{3(2)^{1/3}} = \frac{-}{++} = -$

$f'(7) = \frac{18-5(7)}{3(6-7)^{1/3}} = \frac{18-35}{3(-1)^{1/3}} = \frac{-}{+-} = +$

inc → dec local max at $x = 18/5$, $f(18/5) = \frac{18}{5}\left(6-\frac{18}{5}\right)^{1/3} \approx 6.5$ local max at $(18/5, 6.5)$

dec → inc local min at $x=6$, $f(6) = 6(6-6)^{1/3} = 0$ local min at $(6,0)$

* Note: can use 2nd derivative test to classify max at $18/5$, but doesn't work at $x=6$ as seen in class

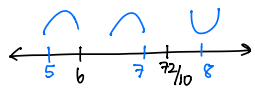
2nd derivative, use quotient rule. $f'(x) = \frac{18-5x}{3(6-x)^{1/3}}$

$f''(x) = \frac{3(6-x)^{1/3}(18-5x)' - (18-5x)\cdot(3(6-x)^{1/3})'}{(3(6-x)^{1/3})^2} = \frac{3(6-x)^{1/3}\cdot(-5) - (18-5x)\cdot 3\cdot\frac{1}{3}(6-x)^{-2/3}\cdot(-1)}{9(6-x)^{2/3}}$

$= \frac{-15(6-x)^{4/3} + 18-5x}{9(6-x)^{2/3}} = \frac{-15(6-x)+18-5x}{9(6-x)^{2/3}} = \frac{-90+15x+18-5x}{(6-x)^{2/3}}\cdot\frac{1}{9(6-x)^{2/3}}$

$f''(x) = \frac{10x-72}{9(6-x)^{4/3}}$ set num, denom. $=0$ $10x-72=0$, $x=\frac{72}{10}$ $9(6-x)^{4/3}=0$, $x=6$

Ex continued



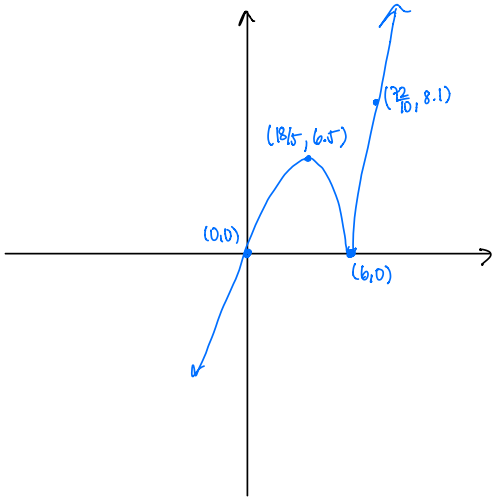
Test points. $f''(5) = \frac{10(5)-72}{9(6-5)^{4/3}} = \frac{50-72}{9(1)^{4/3}} = \frac{-}{++} = -$

$f''(7) = \frac{10(7)-72}{9(6-7)^{4/3}} = \frac{70-72}{6(-1)^{4/3}} = \frac{-}{++} = -$

$f''(8) = \frac{10(8)-72}{9(6-8)^{4/3}} = \frac{80-72}{9(-2)^{4/3}} = \frac{-}{+-} = +$

Inflection point when $\cap$ to $\cup$ at $x = 72/10$, $f(\frac{72}{10}) = \frac{72}{10}\left(6-\frac{72}{10}\right)^{2/3} \approx 8.1$

inflection pt at $(\frac{72}{10}, 8.1)$



$\lim_{x\to\infty} x(6-x)^{2/3} = \infty$ (pos. · pos.)

$\lim_{x\to-\infty} x(6-x)^{2/3} = -\infty$ (neg. · pos.)