Math 33B: Method of Undetermined Coefficients, Eigenvalues and Eigenvectors

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1. Solve $y'' + 2y' - 3y = 5\sin 3t$. (*Hint:* Try a particular solution of the form $y_p = A\cos 3t + B\sin 3t$.)

2. Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$$

3. Consider the matrix

$$B = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}.$$

(a) Using the determinant formula

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix},$$

show that the characteristic polynomial of B is $p(\lambda) = -\lambda^3 + 5\lambda^2 - 8\lambda + 4$.

(b) $p(\lambda) = -\lambda^3 + 5\lambda^2 - 8\lambda + 4$ can be factored as $p(\lambda) = -(\lambda - 2)^2(\lambda - 1)$. What are the eigenvalues of B?

(c) Find the eigenvectors corresponding to the eigenvalues you found in part (b).