Math 33B: Higher Dimensional Systems

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1. Solve $\mathbf{X}' = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \mathbf{X}$.

2. Solve $\mathbf{X}' = \begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{X}$ using

$$\mathbf{X} = c_1\mathbf{v}_1 e^{\lambda t} + c_2(\mathbf{v}_1 t + \mathbf{v}_2)e^{\lambda t} + c_3\left(\mathbf{v}_1\frac{t^2}{2} + \mathbf{v}_2 t + \mathbf{v}_3\right)e^{\lambda t}$$

where $(A - \lambda I)\mathbf{v}_1 = \mathbf{0}, (A - \lambda I)\mathbf{v}_2 = \mathbf{v}_1, (A - \lambda I)\mathbf{v}_3 = \mathbf{v}_2$. *Hint:* You do not need to find the characteristic polynomial. What are the eigenvalues of an upper triangular matrix?

3. Find the general solution to the system

$$\begin{aligned} x_1' &= -\frac{1}{6}x_1 + \frac{1}{6}x_3 \\ x_2' &= \frac{1}{6}x_1 - \frac{1}{3}x_2 \\ x_3' &= \frac{1}{3}x_2 - \frac{1}{6}x_3. \end{aligned}$$

4. Find the solution to the IVP

$$\begin{aligned} x'(t) &= -x(t) + 3y(t) \\ y'(t) &= -3y(t) + 5z(t) \\ z'(t) &= -5z(t) \end{aligned}$$

$x(0) = 0, y(0) = 0, z(0) = 80$. *Hint:* You do not need to find the characteristic polynomial. What are the eigenvalues of an upper triangular matrix?