# Math 31AL Practice Problems III

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1. Calculate the following integrals using geometry:

   (a) $\int_0^4 \sqrt{16 - x^2}\,dx$  (b) $\int_{-3}^3 (3 - |x|)\,dx$  (c) $\int_0^{2\pi} \sin x\,dx$

2. Calculate the following indefinite integrals:

   (a) $\int (2x^3 - 5x^2)\,dx$  (c) $\int x(x-1)\,dx$

   (b) $\int \sec^2 x\,dx$  (d) $\int \left( \cos x + \frac{x-1}{x^3} \right) dx$

3. If $f''(x) = x^3 - 2x + 1$ and $f'(1) = 0$, $f(1) = 4$, find $f'(x)$ and $f(x)$.

4. Evaluate the following integrals using the Fundamental Theorem of Calculus Part I:

   (a) $\int_1^4 x^{3/2}\,dx$  (b) $\int_1^6 |3 - x|\,dx$  (c) $\int_2^3 x^{-2}(1 + x^2)\,dx$

5. Use the Fundamental Theorem of Calculus Part II to evaluate the following derivatives:

   (a) $\dfrac{d}{dx} \int_{1000}^x \sqrt{t}\,dt$  (b) $\dfrac{d}{dx} \int_{-19}^{5x} \dfrac{1}{t}\,dt$  (c) $\dfrac{d}{dx} \int_{\sin x}^{x^2} (t^3 + 1)\,dt$

6. A particle moves with velocity $v(t) = t^2 - 4t + 3$ m/s on an interval from $t = 0$ to $t = 5$ seconds.

   (a) Find the displacement from $t = 0$ to $t = 5$ seconds.
   (b) Find the total distance traveled from $t = 0$ to $t = 5$ seconds (Hint: plot $v(t)$ to see where $v$ is positive and $v$ is negative.)

7. Use the method of substitution to evaluate the following integrals:

   (a) $\int \sin(3x)\,dx$  (c) $\int (x^3 + 1)\sqrt{x^4 + 4x}\,dx$  (e) $\int_0^{\pi/2} \cos x \cos(\sin x))\,dx$

   (b) $\int (4x + 1)^{110}\,dx$  (d) $\int_1^4 \dfrac{\cos \sqrt{t}}{\sqrt{t}}\,dt$

8. Find the average value of $f(x) = x^2$ on the interval $[2, 4]$.

9. Find the area of the region enclosed by $y = x$ and $y = x^2$.

10. Find the volume of the region enclosed by $y = x$ and $y = x^2$ rotated about the $x$-axis.