# Practice Problems III Solutions

Math 31AL
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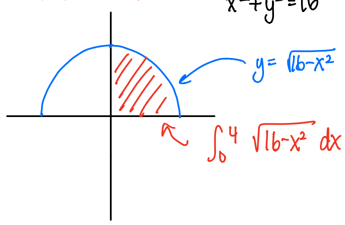
① (a) $\int_0^4 \sqrt{16-x^2}\,dx$

$x=0$ to $x=4$

$y=\sqrt{16-x^2}$
$y^2=16-x^2$
$x^2+y^2=16$ circle w/ center $(0,0)$ and radius 4, so $y=\sqrt{16-x^2}$ represents a semicircle

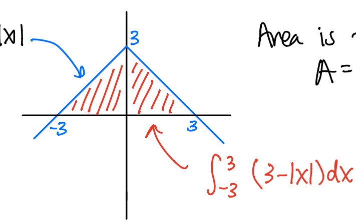



From the picture, area is $\frac{1}{4}$ area of circle w/ radius 4

i.e. $A=\frac{1}{4}\pi r^2=\frac{1}{4}\pi(4)^2=\frac{16\pi}{4}=4\pi$

(b) $\int_{-3}^{3}(3-|x|)\,dx$ There are a couple ways to approach.

One way plot $y=3-|x|$

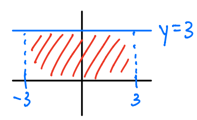


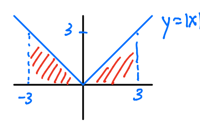

Area is two triangles

$A=\frac{1}{2}bh+\frac{1}{2}bh=\frac{1}{2}(3)(3)+\frac{1}{2}(3)(3)=\frac{9}{2}+\frac{9}{2}$

$=\frac{18}{2}=9$

another way $\int_{-3}^{3}(3-|x|)\,dx=\int_{-3}^{3}3\,dx-\int_{-3}^{3}|x|\,dx$ plot $y=3$ and $y=|x|$



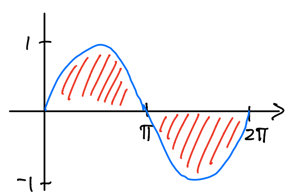




Area is area of rectangle minus two triangles

$A=(6)(3)-\left[\frac{1}{2}(3)(3)+\frac{1}{2}(3)(3)\right]$

$=18-\left(\frac{9}{2}+\frac{9}{2}\right)=18-9=9$

(c) $\int_0^{2\pi}\sin x\,dx$ plot $y=\sin x$




Same area above and below x-axis
So they will cancel out $\rightarrow$ 0

② (a) $\int(2x^3-5x^2)\,dx=\frac{2x^4}{4}-\frac{5x^3}{3}+C=\boxed{\frac{x^4}{2}-\frac{5x^3}{3}+C}$

(b) $\int\sec^2 x\,dx=\boxed{\tan x+C}$

(c) $\int x(x-1)\,dx=\int(x^2-x)\,dx=\boxed{\frac{x^3}{3}-\frac{x^2}{2}+C}$

(d) $\int\left(\cos x+\frac{x-1}{x^3}\right)dx=\int\left(\cos x+\frac{x}{x^3}-\frac{1}{x^3}\right)dx=\int\left(\cos x+x^{-2}-x^{-3}\right)dx=\sin x+\frac{x^{-1}}{-1}-\frac{x^{-4}}{-4}+C$

$=\boxed{\sin x-\frac{1}{x}+\frac{1}{4x^4}+C}$

③ $f''(x) = x^3 - 2x + 1$

$f'(x) = \int (x^3 - 2x + 1)\,dx = \frac{x^4}{4} - \frac{2x^2}{2} + x + C = \frac{x^4}{4} - x^2 + x + C$

But $f'(1) = 0 \rightarrow 0 = \frac{1}{4} - \cancel{1} + \cancel{1} + C \rightarrow C = -\frac{1}{4} \rightarrow \boxed{f'(x) = \frac{x^4}{4} - x^2 + x - \frac{1}{4}}$

$f(x) = \int \left(\frac{x^4}{4} - x^2 + x - \frac{1}{4}\right) dx = \frac{x^5}{20} - \frac{x^3}{3} + \frac{x^2}{2} - \frac{1}{4}x + d$

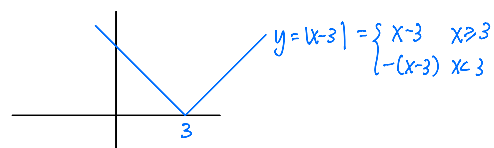
But $f(1) = 4 \rightarrow 4 = \frac{1}{20} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + d \rightarrow 4 = \frac{3}{60} - \frac{20}{60} + \frac{30}{60} - \frac{15}{60} + d \rightarrow 4 = -\frac{1}{30} + d \rightarrow d = \frac{121}{30}$

so $\boxed{f(x) = \frac{x^5}{20} - \frac{x^3}{3} + \frac{x^2}{2} - \frac{1}{4}x + \frac{121}{30}}$

④ (a) $\int_1^4 x^{3/2}\,dx = \frac{x^{5/2}}{5/2}\Big|_1^4 = \frac{2}{5}x^{5/2}\Big|_1^4 = \frac{2}{5}\left(4^{5/2} - 1^{5/2}\right) = \frac{2}{5}\left(\sqrt{4}^5 - \sqrt{1}^5\right) = \frac{2}{5}(2^5 - 1^5) = \frac{2}{5}(32 - 1)$

$= \frac{2}{5} \cdot 31 = \frac{62}{5}$

(b) $\int_1^6 |3 - x|\,dx = \int_1^6 |-1(x-3)|\,dx = \int_1^6 |x-3|\,dx$




$= \int_1^3 |x-3|\,dx + \int_3^6 |x-3|\,dx$

$= \int_1^3 -(x-3)\,dx + \int_3^6 (x-3)\,dx = -\left(\frac{x^2}{2} - 3x\right)\Big|_1^3 + \left(\frac{x^2}{2} - 3x\right)\Big|_3^6$

$= -\left[\left(\frac{9}{2} - 9\right) - \left(\frac{1}{2} - 3\right)\right] + \left(\frac{36}{2} - 18\right) - \left(\frac{9}{2} - 9\right) = -\left(-\frac{9}{2} + \frac{5}{2}\right) + 0 + \frac{9}{2} = \frac{4}{2} + \frac{9}{2} = \frac{13}{2}$

(c) $\int_2^3 x^{-2}(1 + x^2)\,dx = \int_2^3 (x^{-2} + 1)\,dx = \left(\frac{x^{-3}}{-3} + x\right)\Big|_2^3 = \left(\frac{1}{3x^3} + x\right)\Big|_2^3 = \left(\frac{1}{3 \cdot 3^3} + 3\right) - \left(\frac{1}{3 \cdot 2^3} + 2\right) = \frac{1}{81} + 3 - \frac{1}{24} - 2$

$= \boxed{\frac{1}{81} + 1 - \frac{1}{24}}$

⑤ (a) $\frac{d}{dx}\int_{1000}^{x} \sqrt{t}\,dt = \sqrt{x}$

↑ plug in x

(b) $\frac{d}{dx}\int_{-19}^{5x} \frac{1}{t}\,dt = \frac{1}{5x} \cdot (5x)' = \frac{1}{5x} \cdot 5 = \frac{1}{x}$

↑ plug in 5x

(c) $\frac{d}{dx}\int_{\sin x}^{x^2} (t^3 + 1)\,dt = \frac{d}{dx}\left(\int_0^{x^2} (t^3+1)\,dt + \int_{\sin x}^{0} (t^3+1)\,dt\right) = \frac{d}{dx}\left(\int_0^{x^2} (t^3+1)\,dt - \int_0^{\sin x} (t^3+1)\,dt\right)$

$= [(x^2)^3 + 1] \cdot (x^2)' - [(\sin x)^3 + 1] \cdot (\sin x)' = \boxed{(x^6+1) \cdot 2x - (\sin^3 x + 1)\cos x}$

↑ plug in $x^2$ ↑ plug in $\sin x$

⑥ (a) $v(t) = t^2 - 4t + 3$

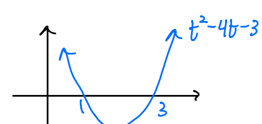
$s = \int_0^5 (t^2 - 4t + 3)\,dt = \left(\frac{t^3}{3} - \frac{4t^2}{2} + 3t\right)\Big|_0^5 = \left(\frac{t^3}{3} - 2t^2 + 3t\right)\Big|_0^5 = \frac{5^3}{3} - 2 \cdot 5^2 + 3 \cdot 5 - \cancel{\left(\frac{0^3}{3} - 2 \cdot 0^2 + 3 \cdot 0\right)}$ 0

$= \frac{125}{3} - 50 + 15 = \frac{125}{3} - 35 = \frac{125}{3} - \frac{105}{3} = \frac{20}{3}$ m

(b) $v(t) = t^2 - 4t + 3 = (t-3)(t-1)$

so $v(t) = 0$ when $t = 3$, $t = 1$

total distance $= \int_0^5 |t^2 - 4t + 3|\,dt$




$= \int_0^1 (t^2 - 4t + 3)\,dt - \int_1^3 (t^2 - 4t + 3)\,dt + \int_3^5 (t^2 - 4t + 3)\,dt$

⑥ (b) continued

$$= \left(\frac{t^3}{3} - 2t^2 + 3t\right)\Big|_0^1 - \left(\frac{t^3}{3} - 2t^2 + 3t\right)\Big|_1^3 + \left(\frac{t^3}{3} - 2t^2 + 3t\right)\Big|_3^5$$

$$= \left(\frac{1}{3} - 2 + 3\right) - \left(\cancelto{0}{\frac{0^3}{3} - 0^2 + 0}\right) - \left[\frac{3^3}{3} - 2\cdot 3^2 + 3\cdot 3 - \left(\frac{1}{3} - 2 + 3\right)\right] + \frac{5^3}{3} - 2\cdot 5^2 + 3\cdot 5 - \left(\frac{3^3}{3} - 2\cdot 3^2 + 3\cdot 3\right)$$

$$= \frac{4}{3} - \left(\cancelto{0}{9 - 18 + 9} - \frac{4}{3}\right) + \frac{125}{3} - 50 + 15 - \left(\cancelto{0}{9 - 18 + 9}\right) = \frac{8}{3} + \frac{20}{3} = \frac{28}{3}\ m$$

⑦ (a) $\int \sin(3x)\,dx \longrightarrow \int \sin u \,\frac{du}{3} = \frac{1}{3}\int \sin u\,du = -\frac{\cos u}{3} + C = \boxed{-\frac{\cos(3x)}{3} + C}$

$u = 3x$ $\quad du = 3\,dx$

$\to dx = \frac{du}{3}$

(b) $\int (4x+1)^{110}\,dx \longrightarrow \int u^{110}\,\frac{du}{4} = \frac{1}{4}\int u^{110}\,du = \frac{1}{4}\,\frac{u^{111}}{111} + C = \boxed{\frac{(4x+1)^{111}}{444} + C}$

$u = 4x+1$ $\quad du = 4\,dx$

$\to dx = \frac{du}{4}$

(c) $\int (x^3+1)\sqrt{x^4+4x}\,dx \longrightarrow \int \sqrt{u}\,\frac{du}{4} = \frac{1}{4}\int u^{1/2}\,du = \frac{1}{4}\,\frac{u^{3/2}}{3/2} + C = \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{1}{6}(x^4+4x)^{3/2} + C}$

$u = x^4 + 4x$ $\quad du = (4x^3 + 4)\,dx$

$du = 4(x^3+1)\,dx$

$\to (x^3+1)\,dx = \frac{du}{4}$

(d) $\int_1^4 \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt \longrightarrow \int_1^2 \cos u \cdot 2\,du = 2\int_1^2 \cos u\,du = 2\sin u\Big|_1^2 = \boxed{2(\sin 2 - \sin 1)}$

$u = t^{1/2}$ $\quad du = \frac{1}{2}t^{-1/2}\,dt$

$du = \frac{1}{2\sqrt{t}}\,dt$

$\to \frac{1}{\sqrt{t}}\,dt = 2\,du$

when $t = 1,\ u = \sqrt{1} = 1$

$t = 4,\ u = \sqrt{4} = 2$

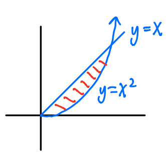
(e) $\int_0^{\pi/2} \cos x \cos(\sin x)\,dx \longrightarrow \int_0^1 \cos u\,du = \sin u\Big|_0^1 = \sin 1 - \sin 0 = \sin 1$

$u = \sin x$ $\quad du = \cos x\,dx$

when $x = 0,\ u = \sin 0 = 0$

$x = \frac{\pi}{2},\ u = \sin\frac{\pi}{2} = 1$

⑧ $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-2}\int_2^4 x^2\,dx = \frac{1}{2}\cdot\frac{x^3}{3}\Big|_2^4 = \frac{x^3}{6}\Big|_2^4 = \frac{4^3}{6} - \frac{2^3}{6} = \frac{64-8}{6} = \frac{56}{6} = \frac{28}{3}$

⑨ 


$x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

intersect at $x = 0,\ x = 1$

$A = \int_0^1 (x - x^2)\,dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{3} - (0 - 0) = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

⑩ same graph above, intersect at $x = 0,\ x = 1$

$V = \pi\int_0^1 \left((x)^2 - (x^2)^2\right)dx = \pi\int_0^1 (x^2 - x^4)\,dx = \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$

$= \pi\left(\frac{1}{3} - \frac{1}{5} - (0 - 0)\right) = \pi\left(\frac{5}{15} - \frac{3}{15}\right) = \frac{2\pi}{15}$