Math 32B: Review "Quiz"

Written by Victoria Kala

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Name: KEY

1. Evaluate the following derivatives:

   (a) $\frac{d}{dx}x^{101} = 101x^{100}$

   * power rule: $\frac{d}{dx}x^n = nx^{n-1}$, $n \neq 0$

   (b) $\frac{d}{dx}(xe^{2x}) = x'e^{2x} + x(e^{2x})'$
   $= \boxed{e^{2x} + 2xe^{2x}}$

   * product rule: $(fg)' = f'g + g'f$

   (c) $(\arcsin(3x))' = \frac{1}{\sqrt{1-(3x)^2}} \cdot (3x)'$
   $= \frac{3}{\sqrt{1-9x^2}}$

   * Chain rule: $(f(g(x)))' = f'(g(x))\,g'(x)$
   * $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$

2. Evaluate the following integrals:

   substitution

   (a) $\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx = \int -\frac{1}{u}\, du = -\ln|u| + C = \boxed{-\ln|\cos x| + C}$ or $\boxed{\ln|\sec x| + C}$

   $u = \cos x$
   $du = -\sin x\, dx$

   (b) $\int \frac{1}{x}dx = \boxed{\ln|x| + C}$

partial fraction decomposition

(c) $\displaystyle\int \frac{1}{4-x^2}dx$

$$\frac{1}{4-x^2} = \frac{A}{2-x} + \frac{B}{2+x}$$

Clear the denominators:

$$(4-x^2)\left(\frac{1}{4-x^2}\right) = \left(\frac{A}{2-x} + \frac{B}{2+x}\right)(4-x^2)$$

$$1 = A(2+x) + B(2-x)$$

$x=-2$: $1 = 4B \rightarrow B = 1/4$

$x=2$: $1 = 4A \rightarrow A = 1/4$

$$\int\left(\frac{1/4}{2-x} + \frac{1/4}{2+x}\right)dx$$

$$\boxed{-\frac{1}{4}\ln|2-x| + \frac{1}{4}\ln|2+x| + C}$$

(d) $\displaystyle\int \frac{1}{x^2+1}dx \quad = \boxed{\arctan x + C}$

* integration by parts

$\int u\,dv = uv - \int v\,du$

(e) $\displaystyle\int x\cos x\ dx \quad = x\sin x - \int \sin x\,dx = \boxed{x\sin x + \cos x + C}$

$u = x$ $\quad dv = \cos x\,dx$

$du = dx$ $\quad v = \sin x$

(f) $\displaystyle\int \left(\sin x - e^{3x} + \frac{1}{x^3} + 5x^2\right) dx$

x^{-3}

$$= -\cos x - \frac{e^{3x}}{3} + \frac{x^{-2}}{-2} + \frac{5}{3}x^3 + C$$

$$= \boxed{-\cos x - \frac{1}{3}e^{3x} - \frac{1}{2x^2} + \frac{5}{3}x^3 + C}$$