Math 33A – Week 1

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Name: KEY

1. Consider the system

$$2x - y = 7$$
$$-x + \frac{1}{2}y = 10$$

(a) Solve the system using elimination or substitution.

elimination: $2x - y = 7$

(multiply 2nd equation by 2) $-2x + y = 20$

$0 + 0 = 27$

$0 = 27$ no solution

(b) How many solutions does this system have? none

(c) Solve using Gaussian or Gauss-Jordan elimination and show you have the same result as part (a).

$$\left(\begin{array}{cc|c} 2 & -1 & 7 \\ -1 & \frac{1}{2} & 10 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} -1 & \frac{1}{2} & 10 \\ 2 & -1 & 7 \end{array}\right) \xrightarrow{2R_1 + R_2 \to R_2} \left(\begin{array}{cc|c} -1 & \frac{1}{2} & 10 \\ 0 & 0 & 27 \end{array}\right)$$

$$\begin{array}{ccc} -2 & 1 & 20 \\ 2 & -1 & 7 \\ \hline 0 & 0 & 27 \end{array}$$

this last row says $0 = 27$ $\Rightarrow$ no solution, same as (a)

2. Consider the system

$$x + 2y = 13$$
$$4x + 8y = 52$$

(a) Solve the system using elimination or substitution.

elimination: $-4x - 8y = -52$

(multiply 1st equation by -4) $4x + 8y = 52$

$0 + 0 = 0$

$0 = 0$ infinitely many solutions

Let $y = t \longrightarrow x + 2t = 13$

$\boxed{\begin{array}{l} x = 13 - 2t \\ y = t \end{array}}$ where $t \in \mathbb{R}$

(b) How many solutions does this system have? infinitely many

(c) Solve using Gaussian or Gauss-Jordan elimination and show you have the same result as part (a).

$$\left(\begin{array}{cc|c} 1 & 2 & 13 \\ 4 & 8 & 52 \end{array}\right) \xrightarrow{-4R_1 + R_2 \to R_2} \left(\begin{array}{cc|c} 1 & 2 & 13 \\ 0 & 0 & 0 \end{array}\right)$$

$$\begin{array}{ccc} -4 & -8 & -52 \\ 4 & 8 & 52 \\ \hline 0 & 0 & 0 \end{array}$$

This last row says $0 = 0 \Rightarrow$ infinitely many solutions

Let $y = t$

First row: $x + 2y = 13 \longrightarrow$ $\boxed{\begin{array}{l} x = 13 - 2t \\ y = t \end{array} \text{ or } \begin{pmatrix} 13 - 2t \\ t \end{pmatrix} \text{ where } t \in \mathbb{R}}$

Same as (a)

3. Consider the system

$$x + 3y = 21$$
$$5x - y = -7$$

(a) Solve the system using elimination or substitution.

elimination: (multiply 2nd row by 3)

$$\begin{aligned} x + 3y &= 21 \\ 15x - 3y &= -21 \\ \hline 16x &= 0 \\ x &= 0 \end{aligned}$$

$0 + 3y = 21$

$y = 7$

$$\boxed{\begin{aligned} x &= 0 \\ y &= 7 \end{aligned}}$$

(b) How many solutions does this system have? one

(c) Solve using Gaussian or Gauss-Jordan elimination and show you have the same result as part (a).

$$\left(\begin{array}{cc|c} 1 & 3 & 21 \\ 5 & -1 & -7 \end{array}\right) \quad -5R_1 + R_2 \to R_2 \quad \left(\begin{array}{cc|c} 1 & 3 & 21 \\ 0 & -16 & -112 \end{array}\right) \quad -\tfrac{1}{16}R_2 \to R_2 \quad \left(\begin{array}{cc|c} 1 & 3 & 21 \\ 0 & 1 & 7 \end{array}\right) \quad -3R_2 + R_1 \to R_1$$

$$\begin{array}{ccc} -5 & -15 & -105 \\ 5 & -1 & -7 \end{array} \qquad \begin{array}{ccc} 0 & -3 & -21 \\ 1 & 3 & 21 \\ \hline 1 & 0 & 0 \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 7 \end{array}\right) \quad \boxed{\begin{aligned} x &= 0 \\ y &= 7 \end{aligned} \text{ or } \begin{pmatrix} 0 \\ 7 \end{pmatrix}} \quad \text{same as (a)}$$

4. (a) Is it possible for a system of linear equations to have exactly two solutions? no

(b) How many solutions can a linear system have? (*Hint*: Three possibilities)

0, 1, or ∞ many

(c) Using Gaussian or Gauss-Jordan elimination, how do you know when you have no solution?

0 = nonzero #

(d) Using Gaussian or Gauss-Jordan elimination, how do you know when you have infinitely many solutions?

* row of zeros (in square systems, not necessarily true in nonsquare systems)

* column with "missing pivot"

pivot is leading entry in row