

1. In \mathbb{R}^2 , show that any reflection about a line L which runs through the origin can be represented as a combination of a rotation and a reflection about x -axis. (you can also try to say it geometrically)
2. In \mathbb{R}^2 , show that the combination of two rotations is a rotation.
3. In \mathbb{R}^2 , show that a rotation and a reflection are not always exchangeable, but two rotations are always exchangeable. (Here the exchangeable means you can change the order of two operators but the result is the same.)
4. Calculate:

(a)

$$\begin{bmatrix} 2 & 4 & 1 & 7 \\ 3 & 2 & -1 & -1 \\ 4 & -2 & 5 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 1 & -7 \\ -2 & 9 \\ 2 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 3 & 5 \\ -1 & 2 & -4 \end{bmatrix} \begin{bmatrix} 6 & 7 & 8 \\ 2 & 1 & 0 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 5 & -2 \\ 2 & 6 \end{bmatrix}$$

5. Consider the matrix

$$A = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & \frac{1}{2} \end{bmatrix}$$

- (a) Calculate A^2 .
- (b) Can you find a matrix B such that $A = B^2$?
- (c) If

$$A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & -\frac{1}{2} \end{bmatrix},$$

do (a) and (b) again.