33A

ask questions in chat

True or False Are all invertible matrices diagonalizable?

False

(2x2 matrix one eigenvector)

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \longleftarrow \lambda = 1 \text{ but only one eigenvector}$$

not diagonalizable

is invertible because $\det A = 1 \neq 0$

invertible :
- 0 is not an eigenvalue
- $\det A \neq 0$

diagonalizable : $n \times n$ matrix, n eigenvectors

Q: Are all orthogonal matrices diagonalizable?

No $\quad A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

orthogonal means $A^TA = I$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

A is orthogonal

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

not diagonalizable over $\mathbb{R}$

Q: $\det(A+B) = \det(A) + \det(B)$?

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\det A = 1 \qquad \det B = 0 \qquad \det A + \det B = 1$

$$A+B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$\det(A+B) = 2$

not the same

5.4 #6 $\quad \operatorname{im}(A) \overset{?}{=} \operatorname{im}(AA^T)$

$\operatorname{im}(A)^\perp \overset{?}{=} \operatorname{im}(AA^T)^\perp$

$\ker A^T \overset{?}{=} \ker((AA^T)^T)$

$\ker A^T \overset{?}{=} \ker(AA^T)$

because $\operatorname{im} A^\perp = \ker A^T$

$B = A^T$ $\quad \ker B \overset{?}{=} \ker(B^T B) \leftarrow$ Theorem 5.4.2(a)

Proof. by Theorem 5.4.2

$$(\ker A^T)^{\perp} \overset{?}{=} (\ker AA^T)^{\perp}$$

$$\operatorname{im} A = \operatorname{im} AA^T \checkmark$$

<u>Midterm 2 #1c</u> $\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} : x_1 + 2x_2 - x_4 = 0 \right\}$

What is dimension?

$x_1 = -2x_2 + x_4$

$$\begin{pmatrix} -2x_2 + x_4 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} x_4$$

dim = 3

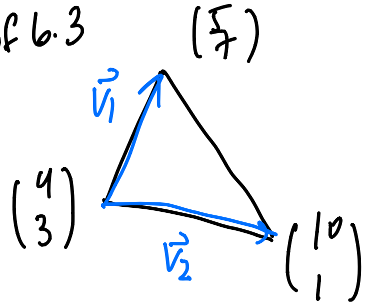
<u>Midterm 2 #1d</u> yes

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \leftarrow \text{projection onto } xy \text{ plane}$$

Area of parallelogram $= \left| \det \begin{pmatrix} | & | \\ \vec{v}_1 & \vec{v}_2 \\ | & | \end{pmatrix} \right|$

Area of trigle $= \frac{1}{2}\left|\det\begin{pmatrix}\vec{v}_1 & \vec{v}_2 \\ 1 & 1\end{pmatrix}\right|$

#3 of 6.3



$$\vec{v}_1 = \begin{pmatrix}5\\7\end{pmatrix} - \begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix}10\\1\end{pmatrix} - \begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}6\\-2\end{pmatrix}$$

$$\frac{1}{2}\left|\begin{vmatrix}1 & 6\\ 4 & -2\end{vmatrix}\right| = \frac{1}{2}\left|(-2-24)\right| = \left|\frac{-26}{2}\right| = |-13|$$

$$= 13$$

Midtrm 2 #3a $\mathcal{B} = \left\{\begin{pmatrix}-3\\2\end{pmatrix}, \begin{pmatrix}-2\\1\end{pmatrix}\right\}$

(a) $\mathcal{B}$-coordinats of $\begin{pmatrix}4\\-3\end{pmatrix}$

$$\longrightarrow \begin{pmatrix}4\\-3\end{pmatrix} = c_1\begin{pmatrix}-3\\2\end{pmatrix} + c_2\begin{pmatrix}-2\\1\end{pmatrix}$$

$$\left(\begin{array}{cc|c}-3 & -2 & 4\\ 2 & 1 & -3\end{array}\right) \xrightarrow{R_1+R_2\to R_1} \left(\begin{array}{cc|c}-1 & -1 & 1\\ 2 & 1 & -3\end{array}\right) \xrightarrow{2R_1+R_2\to R_2}$$

$$\left(\begin{array}{cc|c}-1 & -1 & 1\\ 0 & -1 & -1\end{array}\right) \quad \left(\begin{array}{cc|c}1 & 1 & -1\\ 0 & 1 & 1\end{array}\right) \xrightarrow{R_1-R_2\to R_2} \left(\begin{array}{cc|c}1 & 0 & -2\\ 0 & 1 & 1\end{array}\right)$$

$$\boxed{\begin{matrix}c_1 = -2\\ c_2 = 1\end{matrix}}$$

Check: $-2\begin{pmatrix}-3\\2\end{pmatrix} + 1\begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}6-2\\-2+1\end{pmatrix} = \begin{pmatrix}4\\-3\end{pmatrix}$ ✓

$$\boxed{\vec{w}_{\mathcal{B}} = \begin{pmatrix}-2\\1\end{pmatrix}}$$

(b) $T(\vec{x}) = \underbrace{\begin{pmatrix} 5 & 6 \\ -2 & -2 \end{pmatrix}}_{A} \vec{x}$ $\qquad S = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}$

$\begin{array}{cc} \uparrow & \uparrow \\ \vec{v}_1 & \vec{v}_2 \end{array}$

$$\boxed{B = S^{-1}AS} = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}^{-1} \underbrace{\begin{pmatrix} 5 & 6 \\ -2 & -2 \end{pmatrix}\begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}}$$

$$= \frac{1}{\underbrace{-3+4}_{1}} \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} -15+12 & -10+6 \\ 6-4 & 4-2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} -3 & -4 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} -3+4 & -4+4 \\ 6-6 & 8-6 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}}$$

$$B = S^{-1}AS$$

$$SB = AS$$

$$SBS^{-1} = A$$

Midterm 2 #5 $\quad c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0} \Rightarrow c_1, c_2, c_3 = 0$

$$d_1(\vec{v}_1 + \vec{v}_2) + d_2(\vec{v}_2 - 2\vec{v}_3) + d_3(\vec{v}_1 + \vec{v}_3) = \vec{0}$$

are $d_1, d_2, d_3 = 0$?

$$\underbrace{(d_1 + d_3)}_{c_1}\vec{v}_1 + \underbrace{(d_1 + d_2)}_{c_2}\vec{v}_2 + \underbrace{(-2d_2 + d_3)}_{c_3}\vec{v}_3 = \vec{0}$$

$$\begin{aligned} d_1 \quad\;\; + d_3 &= 0 \\ d_1 + d_2 \quad\;\; &= 0 \\ -2d_2 + d_3 &= 0 \end{aligned} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right)$$

Solve for d_1, d_2, d_3 $\to$ are they all $= 0$?

yes $\to$ L.I.

no $\to$ L.D.

#5.2.37

$$M = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 0 & 5 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 9 \\ 3 & -1 \\ 3 & -1 \\ 3 & 9 \end{pmatrix}$$

last 2 columns don't matter

Find QR factorization

$$\boxed{\underbrace{\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \\ 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}}_{Q}\underbrace{\begin{pmatrix} 3 & 4 \\ 0 & 5 \end{pmatrix}}_{R}}$$

Check $Q^TQ = I$

$$\begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \\ 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

If $\vec{e}_1, \dots, \vec{e}_n$ are eigenvectors then A must be diagonal.

n vectors $\Rightarrow$ A is diagonalizable

$n \times n$

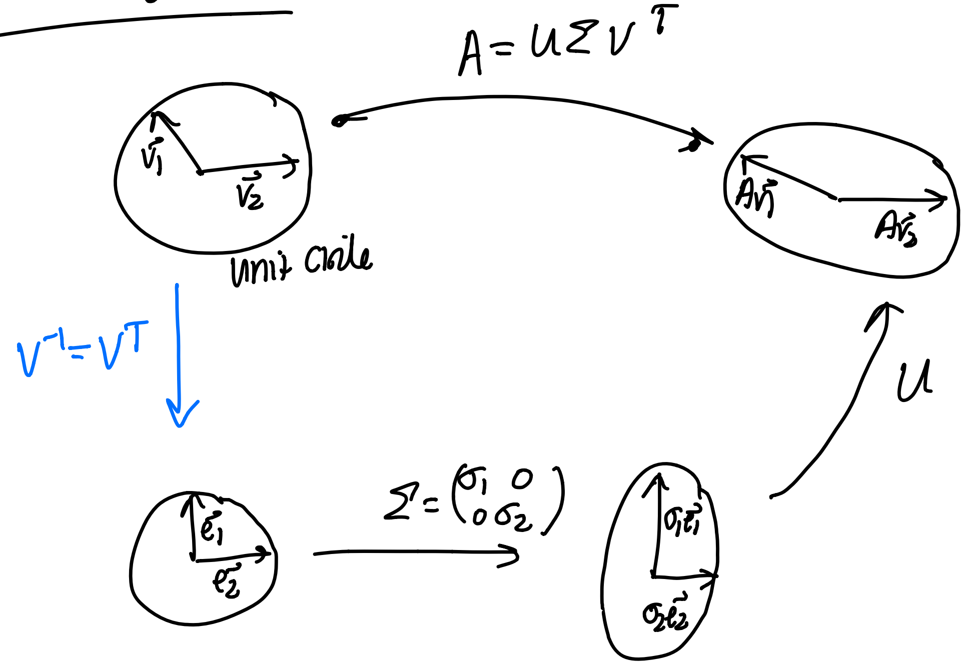
$$A = \begin{pmatrix} | & | & & | \\ \vec{e}_1 & \vec{e}_2 & \cdots & \vec{e}_n \\ | & | & & | \end{pmatrix}\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}\begin{pmatrix} \vec{e}_1 & \vec{e}_2 & \cdots & \vec{e}_n \end{pmatrix}^{-1}$$

$$\overbrace{\begin{pmatrix}1&&&\\&1&&\\&&\ddots&\\&&&1\end{pmatrix}}^{P}\overbrace{\begin{pmatrix}\lambda_1&&\\&\ddots&\\&&\lambda_n\end{pmatrix}}^{D}\overbrace{\begin{pmatrix}1&&\\&\ddots&\\&&1\end{pmatrix}^{-1}}^{P^{-1}}$$

$$\underbrace{\begin{pmatrix}1&&&\\&1&&\\&&\ddots&\\&&&1\end{pmatrix}}_{I}\underbrace{\begin{pmatrix}\lambda_1&&\\&\ddots&\\&&\lambda_n\end{pmatrix}}_{D}\underbrace{\begin{pmatrix}1&&\\&\ddots&\\&&1\end{pmatrix}}_{I}$$

$$= \begin{pmatrix}\lambda_1&&\\&\ddots&\\&&\lambda_n\end{pmatrix} \checkmark \quad A \text{ is diagonal}$$

Pg 409 graph



Midterm 2 #4

$$\operatorname{im}(M)^{\perp} = \{\vec{v} : 0 = \vec{v}\cdot\vec{w},\ \vec{w} \in \operatorname{im} A\}$$

$$\operatorname{im} M = \left\{\begin{pmatrix}2\\-2\\1\\0\end{pmatrix}, \begin{pmatrix}-1\\-1\\-1\\2\end{pmatrix}\right\}$$

$$\vec{v} \cdot \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix} = 0 \quad \vec{v} \cdot \begin{pmatrix} -1 \\ -1 \\ -1 \\ 2 \end{pmatrix} = 0$$

$$2v_1 - 2v_2 + v_3 + 0v_4 = 0$$

$$1v_1 - v_2 - v_3 + 2v_4 = 0$$

$$\left(\begin{array}{cccc|c} 2 & -2 & 1 & 0 & 0 \\ 1 & -1 & -1 & 2 & 0 \end{array}\right)$$