

Problem 1 Use Green's Theorem to find the area of the ellipse $(x/3)^2 + (y/4)^2 = R^2$, where $R > 0$.

Problem 2 Let \mathcal{S} be portion of the ellipsoid $(x/3)^2 + (y/4)^2 + (z/5)^2 = 1$ lying between the planes $z = 0$ and $z = 1$, oriented with outward-pointing normal vectors.

- (a) Draw \mathcal{S} with its boundary orientation.
- (b) Let F be the vector field $\langle -y, -z, 1 \rangle$. Check that F is the curl of $\langle -y, yz, xz \rangle$.
- (c) Use Stokes' Theorem to compute $\iint_{\mathcal{S}} F \cdot dS$.

Problem 3 Let $F = \langle x^3, y^4, z^2 \rangle$ and let \mathcal{S} be the portion of the surface $z = x^2 - y^2$ lying inside $x^2 + y^2 = 1$, oriented in the positive z -direction. Use Stokes' Theorem to compute

$$\oint_{\partial\mathcal{S}} F \cdot dr.$$

Problem 4 Let $f(x, y, z)$ be an infinitely differentiable function of 3 variables. Let \mathcal{S} be the level surface $f(x, y, z) = 0$.

- (a) Explain why the flux of ∇f through \mathcal{S} must be nonzero, assuming that \mathcal{S} has nonzero surface area.
- (b) Use this to show that the vector field $\langle x, y, z \rangle$ does not have a vector potential.

Problem 5 Let \mathcal{S} be the surface given in spherical coordinates by $\rho = 1 + \sin(\phi)$, oriented outward.

- (a) Sketch the surface \mathcal{S} .
- (b) Use Stokes' Theorem to compute $\iint_{\mathcal{S}} F \cdot dS$, where $F = \langle -4z^3, -2x, -3y^2 \rangle$.