Math 33A − Week 9
Written by Victoria Kala
March 11, 2020

Name: _____

1. Compute the determinant of the following matrices:

    (a) $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$

    (b) $B = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$

    (c) $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

    (d) $D = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 1 & 3 & 5 \end{pmatrix}$

2. For the matrices A, B, C, D given in Exercise 1, do $A^{-1}, B^{-1}, C^{-1}, D^{-1}$ exist?

3. The matrix C in Exercise 1c is an example of a diagonal matrix. What can you deduce about the determinants of diagonal matrices?

4. The matrix D in Exercise 1d is an example of a lower triangular matrix. What can you deduce about the determinants of upper or lower triangular matrices?

5. Complete the following properties:

    (a) $\det(I_n) = $ _____

    (b) $\det(AB) = \det(\underline{\hspace{1cm}})\det (\underline{\hspace{1cm}})$

    (c) $\det(A^T) = \det(\underline{\hspace{1cm}})$

6. If $\det(A)$ is nonzero, what is $\det(A^{-1})$? (Hint: use Exercise 5)

7. For the matrices A, B, C, D given in Exercise 1, find $\det(A^{-1})$, $\det(B^{-1})$, $\det(C^{-1})$, $\det(D^{-1})$ without finding the inverses explicitly.

8. If Q is an orthogonal matrix, what are the possibilities for $\det(Q)$? (Hint: use the definition of an orthogonal matrix and Exercise 5)

9. If $A^3 = 4A$, can we guarantee A is invertible? (Hint: take the determinant of both sides)

10. Find the eigenvalues and eigenvectors of the following matrices:

(a) $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$

(b) $E = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$

11. Fill in the blank: the eigenvalues of a diagonal or triangular matrix are the _____ entries.

12. Using the property in Exercise 11, what are the eigenvalues of C and D given in Exercise 1?

13. Consider the matrix $B = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$.

(a) (*Long) Show that the characteristic polynomial of B is $p(\lambda) = -\lambda^3 + 5\lambda^2 - 8\lambda + 4$.

(b) The characteristic polynomial of B can be factored as $p(\lambda) = -(\lambda - 2)^2(\lambda - 1)$. What are the eigenvalues of B?

(c) Find the eigenvectors of B.

14. Fill in the blank: matrices with _____ eigenvalues are diagonalizable.

15. When is a matrix not diagonalizable?

16. Diagonalize the matrices A, E, B given in Exercises 10, 13.

17. Find A^{2020}, E^{2020}, B^{2020} for the matrices A, E, B given in Exercises 10, 13.